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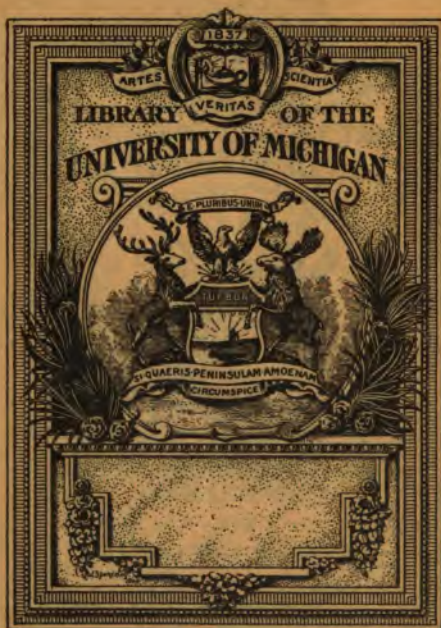
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A PRIMER OF THE
CALCULUS.

BY

E. d. w.
E. SHERMAN GOULD,

M. Am. Soc. C. E.



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PREFACE.

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This work is a development of the infinitesimal calculus as far as the first differentials of algebraic functions of one independent variable and their corresponding integrals. That is to say, it is restricted to the absolute rudiments of the science. Within these narrow limits however, the treatment is tolerably full, and suffices to show how far-reaching a mathematical instrument the Calculus is, even in its elementary steps.

This seems to the author to be the best way of communicating a working knowledge of the science; namely, to teach a few elementary rules and then put them into immediate use, as far as they will go. The student then recognizes the necessity of the more advanced steps.

Very little is said at the start about the logical basis of the science. The few remarks which it seemed necessary to make upon this topic are reserved for the final chapter. It seems wise to the author in any art or science to learn the practical

process first, and the theory, or reason why, afterwards, taking every thing for granted in the mean time, more particularly when, as in the calculus, the theory is somewhat perplexing.

The theory is much better approached after the student's confidence in the practical outcome of the thing has been fully established by actual work. Indeed in this way, he is liable to pick up a good deal of the doctrine as he goes along, and the knowledge of it that he thus acquires will guide him to what he still lacks. The recognized need will point to the best line of investigation.

A number of authorities have been consulted in the preparation of these few pages, while adopting a somewhat different treatment from any of them. Most of the examples are selected.

Those who wish to pursue the study of the calculus in its more advanced developments, will find many admirable text books, both American and English, in which the subject is treated exhaustively.

YONKERS, *April*, 1896.

CHAPTER I.

DIFFERENTIAL CALCULUS.

The differential of the first power of a variable x , which may be considered as representing the varying length of a straight line, is an infinitely small increment, smaller than any quantity that can be expressed in figures, added to such variable. It is expressed thus: dx . This expression is not to be regarded as the product of two factors, but simply as an infinitesimal linear quantity, expressed in terms of the variable.

This quantity dx , is a purely imaginary one, and has no numerical value. If we have, however, two such differentials dx , dy of two variable quantities, x and y for example, the fraction $\frac{dx}{dy}$ indicates an actual ratio, which may receive a numerical value. Later on it will be seen what important practical results are derived from the ratio, $\frac{dx}{dy}$ of two differentials. But it

must be clearly understood from the start that the expressions dx , dy , dz , etc., are merely mathematical figments used as auxiliaries only in *bona fide* calculations.

To convey a more definite conception of this rather indefinite subject, we may, perhaps, borrow an illustration from another science, and say that the differential dx of a straight line represents its indivisible *atom*. Its *molecule* or indefinitely small material particle may be separated by algebraic methods, just as any material substance may be physically divided by mechanical means into indefinitely small parts. But the atom can only be isolated by chemical processes, and is itself intangible and invisible, although ratios may be established between different atoms from which the actual weight of substances may be deduced. In the case of the mathematical atoms, the differential calculus is the solvent, and positive numerical results flow from their mutual relations.

The differential of a variable quantity is the difference between its value after and before the variable has received its in-

infinitesimal increment dx . Thus, if the variable quantity be x^2 , which represents a surface, then x after receiving its increment becomes $x + dx$, and the variable quantity becomes $(x + dx)^2$. Developing, we have $x^2 + 2x dx + dx^2$. Subtracting the primitive value x^2 , from this we have

$$\begin{array}{r} x^2 + 2x dx + dx^2 \\ -x^2 \\ \hline 2x dx + dx^2 \end{array}$$

The term dx^2 is neglected for reasons presently to be stated, and we have for the differential of the variable quantity x^2 the following expression:

$$dx^2 = 2x dx.$$

To understand why dx^2 is neglected, let us take numerical values, and assume $x = 4$ in the above equation. Then $x^2 = 16$. Let us now add the small finite increment 0.10 to x . We then have $(4 + 0.10)^2 = 16 + 0.80 + 0.01$. It will be seen that 0.01 the square of 0.10 is very small in comparison with 0.80. But let us make the increment still smaller, thus:

$$(4 + 0.001)^2 = 16 + 0.008 + 0.000001.$$

The third term has now become not only smaller *actually*, but smaller *relatively* to the second term. The smaller we make the increment, the more insignificant and therefore negligible does its square become. The artifice employed in the calculus consists in supposing the increment dx to be smaller than any expressible quantity. The existence of a still smaller one, of the same kind, its square namely, would be inconsistent with the hypothesis, and it is therefore not only negligible, but must be discarded to conform to the premises.

The expression which we have just been considering may be represented graphically, thus: (Fig. 1) in which x^2 is the primitive area, and $2x dx$ is the differential of x^2 , or the increment of area which corresponds to the increment dx of the variable x . The small square in the right hand upper corner is the discarded infinitely small quantity of the second degree, namely dx^2 .

Without at present going further into this investigation we will at once deduce

the single general rule by which all variable quantities of the class mentioned in the preface may be differentiated, calling attention afterwards to the several special rules which may be formed from it in special cases.

RULE.

MULTIPLY THE EXPRESSION CONTAINING THE VARIABLE BY THE EXPONENT OF THE VARIABLE, DIMINISH THE EXPONENT BY UNITY, AND MULTIPLY BY THE DIFFERENTIAL OF THE FIRST POWER OF THE VARIABLE.*

In applying this rule, the following notation will generally be observed: Roots will be expressed by fractional exponents, and fractions by negative exponents. Thus:

$$\begin{aligned}\sqrt{x} &= x^{\frac{1}{2}} \\ \sqrt[3]{x^2} &= x^{\frac{2}{3}} \\ \frac{1}{x} &= x^{-1} \\ \frac{a}{\sqrt[3]{x^5}} &= a x^{-\frac{5}{3}}\end{aligned}$$

* The words "differential of the *first power* of the variable" are used instead of "the differential of the variable" in order to avoid confusion between the differential of a *variable quantity* and the differential of the *variable*. The differential of the variable quantity, ax , is $2x dx$, the differential of the variable is dx .

As to the negative exponents of fractions it may be remarked, that it amounts to reducing the vulgar fraction to a decimal. Then, since the exponent of a power is its logarithm, its index naturally takes the negative sign.

The above notation is very convenient.

For instance:

$$\sqrt{a} \sqrt[3]{a} = a^{\frac{1}{2}} a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$$

$$\frac{\sqrt{a}}{\sqrt[3]{a}} = a^{\frac{1}{2} - \frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a}$$

$$\frac{\sqrt[3]{a^2}}{\sqrt[3]{a}} = a^{\frac{2}{3} - \frac{1}{3}} = a^{\frac{1}{3}} = \sqrt[3]{a}$$

etc., etc.

EXAMPLES.

$$1.... dx^2 = 2x dx$$

$$2.... d a x^3 = 3ax^2 dx$$

$$3.... \frac{d x^2}{a} = \frac{2x}{a} dx$$

$$4.... d ab \sqrt{y^3} = d ab y^{\frac{3}{2}} = \frac{3 a b y^{\frac{1}{2}}}{2} dy$$

$$5.... d a \sqrt{y} = d a y^{\frac{1}{2}} = \frac{a y^{-\frac{1}{2}} dy}{2} = \frac{a dy}{2 y^{\frac{1}{2}}}$$

$$6.... d \frac{1}{x} = d x^{-1} = -x^{-2} dx = \frac{-dx}{x^2}$$

$$7.... d \frac{\alpha}{y^{\frac{3}{2}}} = d \alpha y^{-\frac{3}{2}} = -\frac{3}{2} \alpha y^{-\frac{5}{2}} dy \\ = -\frac{3 \alpha dy}{2 y^{\frac{5}{2}}}$$

If there are several terms containing variables joined by the sign + or -, the differential of the expression is the algebraic sum of the differentials of each term, separately differentiated. If the expression contains a constant term it disappears in the differentiation, because a constant has no differential. Thus:

$$8.... d \left(ax^2 + by^{\frac{1}{2}} - \frac{ab}{z^{\frac{3}{2}}} \right) = 2ax dx + \frac{b dy}{2 y^{\frac{1}{2}}} \\ + \frac{3 dz}{2 z^{\frac{5}{2}}}$$

$$9.... d (ax^2 + y^3 + b) = 2ax dx + 3y^2 dy.$$

If there are two or more variable factors the differential of the expression is the sum of the differentials of each multiplied by

all the others, that is, the permutation of factors and differentials, thus:

$$10.... d \alpha x y = \alpha y dx + \alpha x dy.$$

$$11.... d \alpha b x^2 y^{\frac{1}{2}} = 2 \alpha b y^{\frac{1}{2}} x dx + \frac{3}{2} \alpha b x^2 y^{-\frac{1}{2}} dy.$$

$$12.... d \frac{x}{y} = d x y^{-1} = \frac{dx}{y} - \frac{x dy}{y^2} \\ = \frac{y dx - x dy}{y^2}$$

$$13.... d \frac{x}{x+y} = \frac{(x+y) dx - x(dx+dy)}{(x+y)^2} \\ = \frac{y dx - x dy}{(x+y)^2}$$

$$14.... d \frac{\alpha}{x^3} + \frac{d y^2}{b} = d \alpha x^{-3} + \frac{d y^2}{b} \\ = \frac{2 y dy}{b} - \frac{3 \alpha dx}{x^4}$$

$$15.... d \frac{-\alpha}{x^{\frac{2}{3}}} = - d \alpha x^{-\frac{2}{3}} = \frac{2 \alpha dx}{3 x^{\frac{5}{3}}}$$

We may now deduce certain special rules for differentiation. From (4) and (5) we see that. *The differential of the square*

root of a variable is the differential of the variable divided by twice the radical.

From (12) and (13) we see that: *The differential of a fraction is the differential of the numerator into the denominator, minus the differential of the denominator into the numerator, divided by the square of the denominator.*

Also from (6) and (3); if the numerator of the fraction is constant *the differential of the fraction is minus the numerator into the differential of the denominator, divided by the square of the denominator*; and if the denominator is constant, *the differential of the fraction is the differential of the numerator divided by the denominator.*

A compound quantity raised to a power may be developed according to its power and each term differentiated as already shown, but it will be easier to treat the quantity in the parenthesis as a simple variable, thus:

$$16.... d(\alpha + bx + cx^2)^5 = 5(\alpha + bx + cx^2)^4 (b + 2cx) dx.$$

$$17.... d \frac{\alpha}{(b^2 + x^2)^3} = - \frac{6 \alpha x dx}{(b^2 + x^2)^4}$$

$$18... d \frac{1}{(1 + x^2)^{\frac{1}{2}}} = - \frac{x dx}{(1 + x^2)^{\frac{3}{2}}}$$

$$19.... d x^3 (\alpha + bx^2)^5 = 3x^3 (\alpha + bx^2)^5 dx + 10 bx^4 (\alpha + bx^2)^4 dx.$$

$$20.... d \frac{x^3}{(1 + x)^3} = \frac{x^2 (3 + x) dx}{(1 + x)^3}$$

So far isolated expressions only have been considered. But these expressions are meaningless as they stand, and to render them complete they must be equated with another variable, thus:

$$y = \alpha + x; y = \alpha - x; y = \alpha x; y = \frac{\alpha}{x}; \\ y = x^m; y = x^{\frac{1}{n}}$$

In such equations the variables, x and y , are said to be functions of each other, because if we attribute different values to one, the value of the other is correspondingly changed. Thus, if when $y = \alpha x$, we attribute a definite value to x , we deduce a corresponding one for y , and vice versa.

If the value of x is arbitrarily assumed, it is said to be the *independent* variable, and y the *dependent* variable, and vice versa.

The above equations, and all others of which they are the types, are differentiated in the usual way, thus:

$$y = (a + x)^3; \quad dy = 3(a + x)^2 dx \quad \dots (1)$$

The second equation (1) is called the *differential equation* of the first. Dividing both sides by dx , gives

$$\frac{dy}{dx} = 3(a + x)^2 \quad \dots \quad (2)$$

The right hand member of this equation (2) is called the *differential coefficient* of the expression. As already mentioned, the differential coefficient plays a most important part in the operations of the calculus, as will be presently seen, but it will be well to say a few words more, in this place, of this matter.

The meaning of such equations as $y = a + x^2$, etc., is simply this: that y is a function of x ; and when we differentiate, and obtain in the above case $dy = 2x \, dx$, we find what infinitesimal change comes over

the function y , when the independent variable x , receives the infinitesimal increment dx .

Thus, in the above example, of which the graphic representation is given by fig. 1, the expression means that y = surface represented by x^2 . The differential equation $dy = 2x dx$ means that when the side of the square receives the infinitesimal increment dx , and becomes $x + dx$, the resulting infinitesimal increment of the surface, dy , is equal to twice the side of the primitive square, multiplied by the increment dx .

The differential co-efficient is in this case, $\frac{dy}{dx} = 2x$, and is the *constant finite value* of the ratio of the infinitesimal increase of surface, to the infinitesimal increase of side. In general: *The differential coefficient is the ratio of the increase of the function to the increase of the variable.*

CHAPTER II.

INTEGRAL CALCULUS.

This is the reverse of a Differential Calculus. To revert to our previous chemical illustration, it is the *crystalization* of the disintegrated atoms, back to their primitive forms.

Integrating a given differential equation consists, therefore, in going back to the primitive function from which it was derived. Thus, if $dy = 2x dx$ is the differential equation of $y = x^2$, then $y = x^2$ is the integral, or primitive function, from which $dy = 2x dx$ was derived.

Evidently, to integrate any expression we might, by "trial and error," find another one which, when differentiated according to previous rules, would reproduce a given expression. But this would be tedious and quite unnecessary. We have only to reverse the rule for differentiation.

Thus, the general rule for the integration of monomials is as follows:

RULE.

INCREASE THE EXPONENT OF THE VARIABLE QUANTITY BY UNITY; DIVIDE BY THE EXPONENT THUS INCREASED AND BY THE DIFFERENTIAL OF THE FIRST POWER OF THE VARIABLE, AND ADD THE SYMBOL C OF A CONSTANT.

The constant is added because in differentiating we drop the constant if there is one. There may or may not be one in the original expression, so we add the symbol C in the existence of a doubt. In any given case the conditions of the problem will show whether or no there is a constant; if there is one, its value can readily be deduced.

The sign of integration is $\int$. Therefore to denote the integration of the differential equation $dy = x^3 dx$, we write :

$$y = \int x^3 dx = \frac{x^4}{4} + C.$$

To determine the fact of there being, or not, a constant, we refer to the original equation. Thus, to integrate the differential equation to the right line, we have $y = \int a \, dx + C$. Then, if the line passes through the origin, $x = 0$ $y = 0$ and consequently $C = 0$. If the line does not pass through the origin, but, when $x = 0$, intersects y at the distance b , from the origin, then $c = b$. This point will be more fully developed later on.

If there are constant factors in the differential equation, they may be put outside of the sign $\int$, for, as they were not affected by the differentiation, neither will they be by the integration. Thus:

$$\int ab \, x^2 \, dx = ab \int x^2 \, dx = \frac{ab \, x^3}{3} + C.$$

The general rule fails only for $dy = \frac{dx}{x}$ for

$$\text{we should have } \int dx \cdot x^{-1} = \frac{x^0}{0} = \frac{1}{0} = \infty$$

This special case enters into the logarithmic functions, which we are not now considering.

EXAMPLES.

$$1. \dots y = a \int x^{\frac{2}{3}} dx = \frac{3a}{5} x^{\frac{5}{3}} + C$$

$$2. \dots y = \int \frac{dx}{x^3} = -\frac{1}{2x^2} + C$$

$$3. \dots y = \int \frac{dx}{x^{\frac{1}{2}}} = 2x^{\frac{1}{2}} + C$$

From (3) we see that: *The integral of the differential of a variable divided by the square root of the variable, is twice the square root of the variable.*

Compound quantities are integrated upon the same principle of reversing the operations of the differential calculus, that is to say, we take the algebraic sum of the separate integrals. Thus, if

$$dy = adx - \frac{b dx}{x^3} + x^{\frac{3}{2}} dx; \text{ we have}$$

$$y = a \int dx - b \int \frac{dx}{x^3} + \int x^{\frac{3}{2}} dx$$

$$y = ax + \frac{b}{2x^2} + \frac{2}{5} x^{\frac{5}{2}} + C$$

If the differential equation be a polynomial of the form $dy = (b x + cx^2)^n$

dx , n being a positive whole number, we may develop the parenthesis according to the indicated power, multiply each time by dx and integrate the separate terms. Thus:

$$\begin{aligned} y &= \int (1 + 2x + 3x^2)^2 dx \\ &= x + 2x^2 + \frac{10}{3}x^3 + 3x^4 + \frac{9}{5}x^5 + C \end{aligned}$$

The expression within the brackets may, however, be more conveniently treated as a simple variable, precisely as was done in the differential calculus, whenever the complete differential of the quantity within the bracket, with or without a constant factor, stands outside of it as a multiplier. In such cases, we can integrate the expression by increasing the exponent of the polynomial by unity, dividing by the exponent so increased, and by the complete differential of the variable quantity within the brackets. This process differs from the general rule for monomials, only in that we divide by the *complete differential*

of the *variable quantity*, instead of only the *first power* of the *variable*.

$$4 \dots \int (a + bx^2)^3 x \, dx = \frac{(a + bx^2)^4}{8b}$$

$$5 \dots 3 \int (ax + x^2)^2 (a \, dx + 2x \, dx) = (ax + x^2)^3$$

$$6 \dots \int (a^2 + x^2)^{\frac{1}{2}} x \, dx = \frac{1}{3} (a^2 + x^2)^{\frac{3}{2}}$$

$$7 \dots \int \frac{x \, dx}{(a^2 + x^2)^{\frac{1}{2}}} = \int x \, dx (a^2 + x^2)^{-\frac{1}{2}} \\ = (a^2 + x^2)^{\frac{1}{2}}$$

$$8. \int (a + bx^n)^m x^{n-1} \, dx = \frac{(a + bx^n)^{m+1}}{n b (m + 1)}$$

CHAPTER III.

APPLICATIONS OF THE DIFFERENTIAL CALCULUS.

Having now learned how to prepare the simpler elements of the calculus, it will be in order to see what practical use can be made of them; what problems unsolvable or solvable only with difficulty by the ordinary processes of algebra and geometry, can be readily overcome by the aid which the calculus brings to their solution.

These problems will present themselves under three forms; those which belong to the differential calculus, those which belong to the integral and those which require a combination of the two. And first, for those which are solved by the differential calculus alone.

PROBLEM.—In the equilateral triangle A, B, C (fig. 2), what is the constant relation between the altitude y and the side x ?

$$y^2 = x^2 - \left(\frac{x}{2}\right)^2 = \frac{3}{4}x^2 \dots \dots (1)$$

Differentiating:

$$2y \, dy = \frac{3}{2} x \, dx$$

$$dy = \frac{3}{4} \frac{x}{y} \, dx$$

The right hand member contains the two variables x and y , and the differential dx . It is necessary to make it homogeneous as regards x and y , and we must therefore express y in terms of x . This can be readily done from (1) which gives $y = \frac{x}{2} \sqrt{3}$.

Therefore —

$$dy = \frac{3}{2} \frac{x}{x \sqrt{3}} \, dx$$

is the differential equation. The differential co-efficient, which expresses the desired ratio is:

$$\frac{dy}{dx} = \frac{\sqrt{3}}{2}$$

This is the constant relation between the infinitely small values dy and dx . But this relation once established between these infinitesimals, holds good for all finite

values also. Therefore, if x should uniformly increase at the rate of 0.10 per second, y would increase at the same time by $\frac{0.10 \sqrt{3}}{2} = 0.0865$.

It is frequently convenient to express this by an equation, using the symbols Δx and Δy denoting finite numerical differences, instead of dx and dy , indicating infinitesimal ones. The above relation would then be:

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{3}}{2}$$

Giving to Δx the definite value 0.10, as above:

$$\Delta y = \frac{0.10 \sqrt{3}}{2} = 0.0865$$

Here is clearly seen the artifice of the calculus, by which when a relation is once established between infinitesimal and uncalculable quantities, these quantities are discarded and their place taken by definite values.

It will not have escaped the observant reader that in this problem there was no

need to have recourse to any new method of calculation, for it was evident at a glance, from the well-known properties of right angled similar triangles that x and y , or any portion of them, would always be in the relation of

$$\frac{y}{x} = \frac{\sqrt{3}}{2}$$

The problem was introduced here, however, for two reasons; first, that the correctness of the results obtained by the processes of the calculus might be easily recognized by applying them to a case admitting of ready verification by ordinary means; and secondly, in order to show the true sphere of the methods which have been studied.

The aid of the calculus was not needed in the above example, because the relations, $\frac{y}{x}$, $\frac{\Delta y}{\Delta x}$; $\frac{dy}{dx}$; were all equal to a *constant numerical quantity*. There was no need of considering the differential, because the differentials were *constants*. Those who pursue the study of the calculus beyond the

limits of this little volume, will see that the fact that the above equation does not admit of a *second differentiation*, shows that the processes of the calculus do not find a fitting field for their exercise, although they lead to a correct result. The province of the calculus is to deal with ratios, which, while constant as between the variables, are constantly changing as to their *numerical values*. But this will be better appreciated from an example.

PROBLEM.--Let the equation to the parabola be

$$y^2 = ax \quad . \quad . \quad . \quad . \quad (1)$$

Differentiating:

$$\begin{aligned} 2 y \, dy &= a \, dx \\ \frac{dy}{dx} &= \frac{a}{2 y} \end{aligned}$$

From (1), $y = a^{\frac{1}{2}} x^{\frac{1}{2}}$. Therefore,

$$\frac{dy}{dx} = \frac{\sqrt{a}}{2 \sqrt{x}} \quad . \quad . \quad . \quad (2)$$

Then (2) is the constant ratio of the infinitesimal increase of the ordinate to the infinitesimal increase of the abscissa, but

it is changing its *numerical value* by infinite gradations.

Let definite values now be assigned in (2). Let $a = 4$, and let it be asked, if x is increasing at the rate of 2 per second, at what rate is y increasing at the precise instant of time when x has become equal to 9. Proceeding as before:

$$\frac{\Delta y}{2} = \frac{1}{3}$$

$$\Delta y = \frac{2}{3}$$

Now, while in the example of the equilateral triangle (Prolem I.) we could get *exactly* the same result more readily by algebraic means, the best we can do in this case is to come *very near* to the relation $\frac{dy}{dx} = \frac{\sqrt{a}}{2\sqrt{x}}$, by a roundabout algebraic process. Thus: Let x in (1) be equal to 9. Then $y = 3\sqrt{a}$. Now give to x the small increment 0.10, making $x' = 9.10$. Then $y' = 3.01662\sqrt{a}$, and

$$\frac{\Delta y}{\Delta x} = \frac{1662}{10000} \sqrt{a} = \frac{\sqrt{a}}{6.017}$$

Take a still smaller increment, 0.01, making $x'' = 9.01$. Then $y'' = 3.001666\sqrt{a}$, and

$$\frac{\Delta y}{\Delta x} = \frac{1666}{10000}\sqrt{a} = \frac{\sqrt{a}}{6.002}$$

The smaller we make the increment the nearer we get to the exact value:

$$\frac{dy}{dx} = \frac{\sqrt{a}}{6}$$

But in the first place we remark, that no general solution is reached (although it might be inferred) for as numerical values are used each case is a special one, and secondly, we can never get the increment quite small enough for an exact solution. By the artifice of the calculus, we assume this increment to be infinitely small—relatively zero, in fact—and then by a process which the explanations of the best authors have failed to make quite clear to the lay mind, a general solution, absolutely exact, is deduced.

The fact of the need of the calculus in the case of the curved line of the parabola, and

not in the case of the straight line of the triangle, may be graphically illustrated. In the case of the equilateral triangle, and all others of which it is the type, the figure (fig. 3) is represented by the equation to the right line, in which the first powers only of the variables x and y are involved. Giving successive equal increments dx, dx , etc., to x , we deduce successive equal increments dy, dy , etc., to y . The relation is constant and equal between all these small increments, and x and y , of the main triangle.

For the curve, however, the case is different (fig. 4). Giving successive equal increments dx, dx etc., to x , we deduce successive *unequal* increments dy, dy^2 etc., to y . The ratios between dx, dy, dy^2 , etc., obey a constant law of change, and are constantly changing their actual, or numerical value.

By means of the calculus this case is dealt with as readily as the former, whereas, dealt with geometrically, an approximation only is reached, and by a tedious process.

EXAMPLES.

1.—If the side of a square increases uniformly at the rate of 3 feet per second, at what rate is its area increasing at the instant when the side = 10 feet?

Representing the surface by y , we have:

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

By the statement:

$$\frac{\Delta y}{3} = 20$$

$$\Delta y = 60 \text{ square feet.}$$

2.—If x increases uniformly at the rate of 1 per unit of time, at what rate does $a + 2x^2$ increase when $a = 4$ and $x = 6$?

$$y = a + 2x^2$$

$$\frac{dy}{dx} = 4x$$

$$\frac{\Delta y}{1} = 4 \times 6$$

$$\Delta y = 24$$

3.—If x increases uniformly at the rate of 1 per unit of time, at what rate does $\frac{x^2}{a}$ increase when $x = 6$ and $a = 4$?

$$\frac{dy}{dx} = \frac{2x}{a}$$

$$\Delta y = \frac{12}{4} = 3$$

4.—The radius of a circular plate of metal is 12 inches. What is the increase of area when the radius is increased by 0.001 inch?

Area of circle, πr^2

$$\frac{dy}{dx} = 2\pi r$$

$$\frac{\Delta y}{0.001} = 2 \times 3.1416 \times 12$$

$$\Delta y = 0.0754$$

This example could also be readily and exactly worked arithmetically, but by calculus the operations are simpler.

5.—A cube of metal, of which the edge

is 12 inches, receives a linear expansion of 0.001. What is its cubical expansion?

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{\Delta y}{0.001} = 3 \times 144$$

$$\Delta y = 0.432$$

Here too, we have a gain over the arithmetical process.

6.—Let the equation to the ellipse be:

$$y^2 = \frac{B^2}{A^2} (A x - x^2)$$

$$\frac{dy}{dx} = \frac{B}{2A} \frac{(A - 2x)}{\sqrt{A x - x^2}}$$

Let $A = 10$; $B = 5$. At what rate is y increasing at the instant when $x = 3$ and $\Delta x = 0.20$ per second?

$$\frac{\Delta y}{0.20} = \frac{5(10 - 6)}{20 \sqrt{30 - 9}}$$

$$\Delta y = 0.0436 \text{ per second.}$$

Our next applications will be very important ones, not only in their practical

bearing upon the problems of coördinate geometry, but also as clearly showing the wonderful power of the calculus as an instrument for analytical investigation.

PROBLEM.—A curve OB (fig. 5) being given by its equation, to find the direction of the tangent at a certain point M , of which the coördinates are OP , PM , by determining the sub-tangent PT , in terms of the abscissa $OP = x$.

It will be instructive to see first how this might be done geometrically, following, however, the same method of reasoning which is adopted in the calculus. Considering the figure geometrically, it is clear that although OB is a curved line, a portion of it, MM' could be taken, so small as not to perceptibly differ from a straight line. If, therefore, we take a small distance PP' , on the "axis of x ," and, either by construction or calculation determine the ordinates PM , $P'M'$, we may consider the hypotenuse MM' of the little right angle triangle $MM'R$, as very nearly representing the little arc of the curve between the

two points M and M'. The solution is now very simple, for from similarity,

$$PT = \frac{MR \times PM}{M'R} \quad . \quad . \quad (1)$$

To test this, take the parabola $y^2 = 4x$, and let $x (= OP) = 9$. Then $y = 6$. Give x the small increment, $PP' = \Delta x = 0.01$. Then $x' = 9.01$, and $y' = 6.0033$. Substituting the values $MR = 0.01$; $M'R = 0.00333$, and $PM = 6$, in (1), we have:

$$PT = 6 \times \frac{0.01}{0.00333} = 18.018$$

The smaller we make Δx , and the smaller, consequently, Δy , the nearer will MM' approach to a straight line, and the nearer will our approximation become, particularly, if as in the present instance, the point M occurs where the curve is naturally flat. Still, it will always be recognized that it is only an approximate answer which is obtained, after all.

Now let us apply the calculus, and assume $dx (= PP')$ to be *infinitely* small,

and dy ($= M'R$) to be so also. We then have, from (1):

$$PT = y \frac{dx}{dy} \dots (2)$$

Let us apply this formula to the parabola $y^2 = ax$ just considered (the coefficient a or 4 is immaterial, since it will disappear). We have:

$$\begin{aligned} 2y \, dy &= a \, dx \\ \frac{dx}{dy} &= \frac{2y}{a} \\ PT &= \frac{2y^2}{a} \end{aligned}$$

But $y^2 = ax$. Therefore

$$PT = \frac{2ax}{a} = 2x$$

If, as in the previous example, we have $x = 9$, then $PT = 18$.

The difference between the geometrical method and that of the calculus is that the latter gives a rapid and exact method of determining the proper ratio of the ele-

ments, by means of the relation $\frac{dx}{dy}$, for any point x, y , of any curve.

EXAMPLE.—Determine the sub-tangent for the ellipse, of which the equation is

$$y^2 = \frac{B^2}{A^2} (A x - x^2) \quad . \quad . \quad (1)$$

$$2 y dy = \frac{B^2}{A^2} (A - 2 x) dx$$

$$\frac{dx}{dy} = \frac{2 A^2 y}{B^2 (A - 2 x)}$$

$$PT = \frac{2 A^2 y^2}{B^2 (A - 2 x)}$$

To express the value of PT wholly in terms of x , substitute the value of y^2 from (1).

$$PT = \frac{2 A^2 B^2 (A x - x^2)}{A^2 B^2 (A - 2 x)} = \frac{A x - x^2}{\frac{1}{2} A - x}$$

The subnormal P Q (fig. 6) may be found in a similar manner from the similarity of the triangles MM' R and M P Q, to be:

$$PQ = y \frac{dy}{dx}$$

This differs from the equation to the sub-tangent only in the reversal of the ratio.

EXAMPLES.

1.—To find the subnormal to the parabola $y^2 = ax$.

$$PQ = \frac{a}{2}$$

2.—To find the subnormal to the ellipse,

$$y^2 = \frac{B^2}{A^2} (Ax^2 - x^2)$$

$$PQ = \frac{B^2}{A^2} \left(\frac{1}{2} A - x \right)$$

We may also find the trigonometrical tangent T to the curve at the point M (fig. 5), in the same manner. For calling PT the radius = R , and MP the desired tangent = Tan we have, from similar triangles:

$$Tan = R \frac{dy}{dx}$$

When $R = 1$,

$$Tan = \frac{dy}{dx}$$

EXAMPLE.—What is the trigonometrical tangent to the parabola $y^2 = ax$?

$$T = \frac{y}{2x}$$

It will be observed that in all these cases the curve has been considered as made up of an infinite number of infinitesimal straight lines. Indeed, it may be said that, in regard to the class of problems which we have been just considering, the office of the calculus has been to enable us, by an artifice, or mathematical figment, to treat curves as if they were straight lines. The solution of the problem then becomes very easy.

CHAPTER IV.

MAXIMA AND MINIMA.

We will now take up a class of problems which, from its importance, merits a separate chapter.

It is in solving problems of maximum and minimum values that the utility of the calculus as a practical instrument of calculation becomes more immediately apparent. The student of the calculus will not be long in perceiving that, apart from the subject we are about to treat, the use of the calculus lies in the direction of investigation, demonstration, and the establishment of formulæ, rather than that of current calculation. The formula once established, by its aid, it is cast aside, like its own "differentials," when once their useful purpose has been accomplished.

In the ascertaining of maximum and minimum quantities, however, the case is different, and there the processes of the calculus are freely employed in the solution of special cases, which either cannot be

solved at all by algebraic methods, or can be only in a more indirect and intricate manner.

When a variable function has a maximum value, it goes on increasing up to a certain point, and then decreases. It passes, therefore, from a positive to a negative state, and to do this, must go through zero. At the instant of time—or point in space—when its growth is neither positive nor negative, but entirely neutral, or zero, it has attained its maximum value. The same is true for minimum values.

These facts bring such questions within easy reach of calculus processes. When a continuously increasing—or decreasing—function ceases to vary it has no differential, or rather its differential becomes zero. In other words, when a quantity is at its maximum or minimum value, it cannot be increased on the one hand or diminished on the other. Its differential, or the infinitesimal quantity by which it might otherwise be increased or diminished, must, consequently, under the hypothesis, be equal to zero. Therefore, we have only to

establish the statement, in algebraic terms, of the condition which it is desired shall be maximum or minimum, differentiate it, make the differential equal to zero, and solve, algebraically, what remains.

To take a very simple illustration: Let it be required to divide a given line into two such parts that their product shall have a maximum value.

Let a = the whole line, and x one of the parts. Then $a - x$ = the other part. By the conditions of the question:

$$x (a - x) = \text{Maximum.}$$

Developing,

$$ax - x^2 = \text{Maximum.}$$

Differentiating and making the differential equal zero:

$$a \, dx - 2x \, dx = 0$$

Dividing both sides by dx :

$$a - 2x = 0$$

$$x = \frac{a}{2}$$

Hence, the line must be divided into equal parts.

EXAMPLES.

1.—Divide a line into two such parts that the sum of their squares shall be minimum.

$$x^2 + (a - x)^2 = \text{Minimum.}$$

$$2x \, dx - 2(a - x) \, dx = 0$$

$$x - a + x = 0$$

$$x = \frac{a}{2}$$

2.—Find a fraction, x , that shall exceed its cube by the greatest possible value.

$$x - x^3 = \text{Maximum.}$$

$$dx - 3x^2 \, dx = 0$$

$$1 - 3x^2 = 0$$

$$x = \sqrt{\frac{1}{3}}$$

It is frequently more convenient to use two unknown quantities.

3.—Among right angle triangles of equal hypotenuse, find that which has the greatest area.

Let a = constant hypotenuse, x = one side of the triangle and y = the other.

We require two equations, which the conditions of the problem give us.

$$x^2 + y^2 = a^2 \quad . \quad . \quad . \quad (1)$$

$$\frac{xy}{2} = \text{Maximum} \quad . \quad . \quad . \quad (2)$$

Differentiating (2):

$$y \, dx + x \, dy = 0$$

$$dx = -\frac{x \, dy}{y}$$

Differentiating (1):

$$x \, dx + y \, dy = 0$$

$$dx = -\frac{y \, dy}{x}$$

Equating the two values of dx

$$\frac{x \, dy}{y} = \frac{y \, dy}{x}$$

$$x^2 = y^2$$

$$x = y$$

This problem can be easily solved geometrically by striking a semi-circle on the given hypotenuse, which must contain all the right angled triangles than can be

constructed upon it. The figure will also be an admirable graphic representation of a maximum value.

4.—It is required to make a cylindrical vessel of given capacity C , with the minimum quantity of sheet tin. What must be the relation between the height h , and the radius of the base r ?

$$\pi r^2 h = C \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\pi r^2 + 2 \pi r h = \text{Minimum.} \quad (2)$$

$$r \, dr + h \, dr + r \, dh = 0$$

$$dh = - \frac{(r + h) \, dr}{r}$$

From (1)

$$dh = - \frac{2h \, dr}{r}$$

$$h = r$$

5.—In the right angled triangular field $A B C$ (fig. 7) lay off the maximum rectangular plot, $x y$, one side, x , to stand upon the base $A B = b$. What must be the re-

lation between the side y , and the height
 $A C = h$?

$$x y = \text{Maximum} \dots (1)$$

From similar triangles:

$$\frac{h - y}{x} = \frac{h}{b} \dots (2)$$

From (1)

$$dx = - \frac{x dy}{y}$$

From (2)

$$dx = \frac{x dy}{y - h}$$

$$\frac{x dy}{y - h} + \frac{x dy}{y} = 0$$

$$y = \frac{h}{2}$$

6.—A man in a boat at B, (fig. 8) wishes to reach the point C in a straight line of coast, as quickly as possible. The distance $A C = 5$ miles; $A B = 3$ miles. He can row 4 miles an hour, and go on foot 5 miles an hour.

At what point D, distant x miles from A, should he land and take to his heels?

$$y^2 - x^2 = 9 \quad \dots \dots (1)$$

$$\frac{y}{4} + \frac{5-x}{5} = \text{Minimum} \dots (2)$$

$$5 \, dy - 4 \, dx = 0$$

$$dy = \frac{4 \, dx}{5}$$

$$\text{From (1) } dy = \frac{x \, dx}{y}$$

Therefore,

$$y = \frac{5x}{4} = \sqrt{9 + x^2}$$

$$\frac{25x^2}{16} = 9 + x^2$$

$$x = 4 \text{ miles.}$$

7.—There are two lights of relative intensity, A and B, distant A B = D from each other. What point P, from which both are visible, distant x from the most powerful light A, receives the least illumination?

As the illumination of a light is inversely

as the square of the distance from it, we have

$$\frac{A}{x^3} + \frac{B}{(D-x)^3} \dots \text{Minimum.}$$

$$\frac{B \, dx}{(D-x)^3} - \frac{A \, dx}{x^3} = 0$$

$$\frac{B}{(D-x)^3} = \frac{A}{x^3}$$

$$x = \frac{D \sqrt[3]{A}}{\sqrt[3]{B} + \sqrt[3]{A}}$$

If $A = 9$; $B = 7$, and $D = 25$ miles,
 x = about 13 miles.

8.—It is required to make up a quantity of lumber, of q square feet, into a box and cover, which shall have the maximum capacity.

We know already, from the first illustration, (that of a line so divided that the product of the parts shall be a maximum,) that the base of the box must be a square. It remains to ascertain the height, h .

$$2x^2 + 4x \, h = q \dots (1)$$

$$x^2 \, h = \text{Maximum} \dots (2)$$

Differentiating (2)

$$dh = - \frac{2 h dx}{x} \dots (3)$$

Differentiating (1):

$$dh = - \left(\frac{x + h}{x} \right) dx \dots (4)$$

Equating (3) and (4)

$$x = h$$

Hence the box must be a cube.

9.—It is desired to illuminate a circle (see fig. 9) of radius R, by an electric light, so that its circumference shall receive the maximum illumination. At what height, x , should the light be hung?

The intensity of illumination at any point in the circumference will be directly as the sine of the angle A, and inversely as the square of the distance A B.

The illuminating effect therefore, at the point A, will be:

$$\frac{\sin A}{A B^2}$$

We have $AB = \sqrt{R^2 + x^2}$, and therefore, by trigonometry Sine A

$$\begin{aligned}
 &= \frac{x}{\sqrt{R^2 + x^2}} \quad \text{Whence:} \\
 \frac{\sin A}{AB} &= \frac{x}{\sqrt{R^2 + x^2} \times (R^2 + x^2)} \\
 &= \frac{x}{(R^2 + x^2)^{\frac{3}{2}}}
 \end{aligned}$$

Differentiating, etc.,

$$(R^2 + x^2)^{\frac{3}{2}} - 3(R^2 + x^2)^{\frac{1}{2}} x^2 = 0.$$

Dividing out by $(R^2 + x^2)^{\frac{1}{2}}$;

$$2x^2 = R^2$$

$$x = \frac{R}{\sqrt{2}} = 0.707 R.$$

If $R = 100$ feet, $x = 71$ feet.

10.—It is required to build a reservoir of capacity = C. It is to be in excavation, with a concrete bottom and rubble masonry side walls. Six inches of concrete at the bottom has been deemed sufficient, but to cover contingencies, and also to allow for the foundations of the retaining

walls, etc., 1 foot thickness is allowed in the calculation. The form is to be square, in plan. Let x = one side of the square, interior measurement. Let the cross section of the retaining walls be equivalent to $\frac{h^2}{3}$, h being the height of wall. Ignoring cross corners, and taking $4x$ as the total length of wall, what should the height, h , be in order to use the smallest volume of masonry, both concrete and rubble?

The capacity, neglecting face batter of wall, will be represented by :

$$x^2 h = C \dots \dots \dots (1)$$

The total volume of masonry, side walls and bottom by:

$$\frac{4xh^2}{3} + x^2 = \text{Minimum} \dots \dots (2)$$

Differentiating (2) etc.

$$4xh dh + 2h^2 dx + 3x dx = 0.$$

$$dh = -\frac{(2h^2 + 3x)dx}{4xh} \dots \dots (3)$$

From (1)

$$dh = -\frac{2h dx}{x} \dots\dots\dots (4)$$

Equating (3) and (4):

$$2h^2 = x$$

$$\text{From (1) } x = \frac{\sqrt[5]{c}}{\sqrt{h}}$$

$$\text{Hence } h = \sqrt[5]{\frac{c}{4}} = 0.76 \sqrt[5]{c}$$

If $c = 1,000,000$ cubic feet, $h = 12$ feet, and $x = 289$ feet, both in round numbers.

Practically, the dimensions of reservoirs are determined by a number of considerations of which the minimum volume of masonry is only one, and generally not the controlling one. It is well however, in designing such work, to figure out the approximate dimensions for minimum volume, as a basis to vary from. The calculation can only be an approximate one, because the depth of foundation and other details of construction vary when the work is put in execution, and the differences in cost of the different classes of work, con-

crete, rubble, etc., enter also as disturbing elements.

11.—A column (Fig. 10) of height h , is surmounted by a statue of height S , the top of which is consequently $h + S = H$, from the ground, which is supposed to be level. At what point A , distant x from the foot of the column, does the statue subtend the greatest angle?

From the data, $AB = \sqrt{H^2 + x^2}$, and $AC = \sqrt{h^2 + x^2}$. It is required that the sine of the angle BAC , which is equal to the line y , drawn from the point C , perpendicular to AB , and divided by AC , shall be a maximum. Therefore,

$$\frac{y}{\sqrt{h^2 + x^2}} = \text{Maximum, ... (1)}$$

From similar triangles,

$$\frac{y}{S} = \frac{x}{\sqrt{H^2 + x^2}}; y = \frac{Sx}{\sqrt{H^2 + x^2}}$$

. Substituting and squaring in (1); we have :

$$\frac{S^2 x^2}{(H^2 + x^2)(h^2 + x^2)} = \text{Maximum.}$$

This will be written in thus:

$$S^2 x^2 (H^2 + x^2)^{-1} (h^2 + x^2)^{-1} = \text{Maximum.}$$

Differentiating the three factors and reducing:

$$x - \frac{x^3}{(H^2 + x^2)} - \frac{x^3}{(h^2 + x^2)} = 0 \dots (2)$$

Finally:

$$x = \sqrt{H h}.$$

This is the point at which the statue appears to the best advantage, and if a circular railing were put around the column, it should have a radius $= \sqrt{H h}$.

It will have been remarked that the same process has been used indiscriminately, in the above examples, for obtaining maximum and minimum values. But this involved no uncertainty, because an examination of the data, would in all cases readily show what the converse of the proposition would be. Thus, in example (7) when a minimum value was required, it would be evident that the position of maximum illumination would be as close as possible to the more powerful light.

Other more complicated cases occur where it is necessary to ascertain whether there is, or can be, maximum or minimum values, or if there be a recurring series of such values. These questions are successfully treated by more advanced processes than those already investigated, but even by the resources already acquired, we are able to solve a large class of questions either out of the reach of algebraic methods, or only attainable by them, after tedious and intricate processes.

In all cases, having obtained a maximum or minimum value, its correctness should be checked by taking numerical values a little greater and a little less than the value obtained, and comparing them with it.

CHAPTER V.

APPLICATIONS OF THE INTEGRAL CALCULUS.

By means of the Integral Calculus we readily solve many problems relating to the quadrature of curves, and the cubature of solids of revolution.

PROBLEM.—Let it be required to find the area of the plane curve OB (fig. 11) referred to rectangular co-ordinates.

Let $PM = y$ be any ordinate of the curve, and $OP = x$, the corresponding abscissa. Let $PP' = dx$ be the differential of x , or the infinitesimal increment added to it, and $M'R = dy$ the corresponding infinitesimal increment of y . Then $PP'MM'$ is the corresponding infinitesimal increment to the area of the curve, and since dx and dy are infinitely small, MM' will be so also, and will not differ from a straight line. The figure $PP'MM'$ is therefore a trapezoid, and its area is:

$$\left(\frac{y + y + dy}{2} \right) dx = y dx + \frac{dy dx}{2}$$

But $dy \, dx$ is an infinitesimal of the second order, consequently $\frac{dy \, dx}{2}$, which represents the area of the small triangle $MM' R$ must be dropped, and we have the equation:

$$dA = y \, dx \dots (1)$$

In the above equation, dA is the infinitely small area $PP' MM'$. But the whole area is made up of the sum of these infinitesimal rectangles. Consequently the total area, A , is represented by the integral of the above differential equation, thus:

$$A = \int y \, dx + C \dots (2)$$

This equation is solved for any given curve of which we know the equation, by finding the value of y , in terms of x , or of dx , in terms of y , and then integrating.

Thus, to find the area of the parabola $y^2 = px$. Deducing the value of y , and inserting in (1);

$$\begin{aligned} dA &= p^{\frac{1}{2}} x^{\frac{1}{2}} dx \\ A &= p^{\frac{1}{2}} \int x^{\frac{1}{2}} dx = \frac{2p^{\frac{1}{2}}}{3} x^{\frac{3}{2}} + C \end{aligned}$$

This may be written in terms of x and y , thus:

$$A = \frac{2}{3} x y + C$$

That is the area of the parabola is two-thirds the circumscribing rectangle $x y$.

To determine the constant, when $x = 0$, it is plain that the area, and therefore the constant, are also zero. Hence, there is no constant in this case.

Generally we wish to find the area of the curve between limits, or between certain values of x , a and b , for instance.

The equation is then written:

$$A = p^{\frac{1}{3}} \int_a^b x^{\frac{2}{3}} dx$$

In solving the integral, we make $x = b$, and $x = a$, successively, in the equation

$A = \frac{2}{3} p^{\frac{1}{3}} x^{\frac{3}{2}} + C$ and subtract one result from the other. Thus:

$$\frac{2}{3} p^{\frac{1}{3}} b^{\frac{3}{2}} + C - \left(\frac{2}{3} p^{\frac{1}{3}} a^{\frac{3}{2}} + C \right) = \frac{2}{3} p^{\frac{1}{3}} (b^{\frac{3}{2}} - a^{\frac{3}{2}})$$

If we take the value of dx in terms of dy , we have:

$$2 y dy = p dx; dx = \frac{2 y dy}{p}$$

$$d A = \frac{2 y^2 dy}{p}$$

$$A = \frac{2}{p} \int y^2 dy$$

$$A = \frac{2}{3} \frac{y^3}{p} \times y = \frac{2}{3} xy$$

In establishing the above formulæ, we have made use of the differential calculus in preparing the elements, and (2) contains the differential dx . But in practice, it is not necessary to multiply and divide by dx , and we simply write the value of y in terms of x , from the equation of the curve and then perform the integration, ignoring dx . For instance, to find the area of the curve.

$$y = x^3 + ax^2$$

We simply integrate the right hand

member as it stands, omitting the multiplication and division by dx ; thus:

$$A = \frac{x^4}{4} + \frac{ax^3}{3} + C = \frac{x^3}{12} (3x + 4a) + C$$

In the above, when $x = 0$, $A = 0$.

$$\text{When } x = a, A = \frac{7}{12} a^4$$

EXAMPLES.

1.—Find the area of $y = x^3 - bx^2$ between the limits $x = 0$, $x = b$. *Ans.* $\frac{b^4}{12}$

2.—Find the area of $y = 5 + (x - 3)^{\frac{1}{2}}$ between $x = 3$, $x = 12$. *Ans.* 6 3

In these problems also, we can get an approximate solution by algebraic processes, from the equation to the curve, working on the same general lines as those followed in the calculus.

Thus, to calculate the area of the parabola $y^2 = x$, between the limits of $x = 1$ and $x = 4$. Give to $x = 1$ successive small equal increments of say, 0.50, and calculate the corresponding values of y .

Then add the values of y , and multiply the sums by 0.50. Thus:

$x = 1.0$		$y = 1.00$
$= 1.5$		$= 1.22$
$= 2.0$		$= 1.41$
$= 2.5$		$= 1.58$
$= 3.0$		$= 1.73$
$= 3.5$		$= 1.87$
$= 4.0$		$= 2.00$
$10.81 \times 0.50 = 5.405$		

The true value, by calculus, is $4\frac{1}{2}$. By taking the increment very much smaller, a much nearer approximation to a correct result would be obtained. But we would never get the increment quite small enough, and the smaller it is, the greater the arithmetical labor of calculating the successive small areas and adding them up. By calculus, we assume the value of $y \, dx$ to be infinitesimal, and then by a few rapid transformations, re-integrate them. Moreover, the solutions of the calculus are general for each regular curve, and result in a formula which can be used for all curves

of the same class without further calculation. For instance, we have found that for the common parabola the area is $\frac{2}{3}xy$.

Therefore in the example just worked out algebraically, where $x = 4$ and $y = 2$, the area is $\frac{16}{3}$. When $x = 1$ and $y = 1$, the

area is $\frac{2}{3}$. Between these two values of x ,

the area of the curve is $\frac{16 - 2}{3} = 4\frac{2}{3}$.

In the numerical example just given, the problem was worked out by the same process as that employed by the calculus, for the sake of the comparison. It is worthy of note, however, that by calculating values of y for intermediate values of x , we should, in many cases, easily rival the accuracy of the calculus by algebraic methods, although at the cost of greatly increased labor. Thus, in the example referred to, when it was desired to calculate the area of the curve $y^2 = x$ between the limits of $x = 1$; $x = 4$, we would calculate our first value of y for $x = 1.25$, instead of 1, and

advance to $x = 3.75$, instead of 4, as follows:

$x = 1.25$	$y = 1.118$
1.75	= 1.323
2.25	= 1.500
2.75	= 1.658
3.25	= 1.803
3.75	= 1.936

9.338

Multiplying by 0.50, gives for the desired area, 4.669, almost exactly the result given by the calculus, and by taking an increment still smaller than 0.50, the result would have been still nearer. This curve, $y^2 = x$, is a very flat one, between the assumed limits. The sharper the curve, the smaller must be the increment in any algebraic or geometric calculation of areas.

Areas of curves may sometimes be found in a somewhat irregular way. For instance, Mr. Weston, in his "Simple Explanations of Engineering Formulæ," gives the following ingenious determination of the area of a circle, which otherwise is rather complicated, and cannot be reached by rules

already given. The process may be thus explained:

Conceive the circle (fig. 12) to be divided into an infinite number of concentric rings, or zones. The circumference of each one of these is $2x\pi$, x being the radius of any one of the concentric circles, and the area of each, $2x\pi \, dx$, dx being the infinitely narrow width of each. We have then, integrating between $x = 0$, $x = r$:

$$dA = 2x\pi \, dx.$$

$$A = 2\pi \int_0^r x \, dx$$

$$A = \pi r^2$$

PROBLEM.—Let it be required to calculate the volume of a solid, engendered by the revolution of a curve of given equation, about one of its axes.

The solid is conceived to be divided up into an infinite number of infinitely thin slices, taken at right angles to the axis of x , and their common differential equation integrated.

Thus, (fig. 13) the area of any one of the thin slices is the area of the circle described

by y around the axis $o x$, and the differential of the volume of the solid will be:

$$dV = \pi y^2 dx \dots (1)$$

EXAMPLES.

1.—Find the volume of the paraboloid of which the generating curve is $y^2 = ax$.

$$\begin{aligned} dV &= a \pi x dx \\ V &= a \pi \int x dx \\ &= \frac{a \pi x^2}{2} + C \end{aligned}$$

To express this equation in terms of x

and y , put $x = \frac{y^2}{a}$. Then:

$$V = \frac{\pi x y^2}{2}$$

Hence, the volume of all common paraboloids is half that of the circumscribing cylinder.

2.—Find the volume of the cone engendered by the revolution of the right angled triangle, $y = ax$, around the axis of x .

$$\begin{aligned}
 y^2 &= a^2 x^2 \\
 dV &= \pi a^2 x^2 dx \\
 V &= \pi a^2 \int x^2 dx \\
 &= \frac{\pi a^2 x^3}{3}
 \end{aligned}$$

Suppose now it were desired to find the volume of a solid other than of revolution; a pyramid, for instance, with a square, triangular or other base. Of course, it would not be necessary to recur to the calculus for this, because the problem can be readily solved by solid geometry, as indeed can that of the cone just considered, also. But it will be instructive to see how it would be necessary to proceed by the calculus.

It will have been noted that, for the calculation of both areas and solids, it was necessary to know the equation of the figure involved. That being given, we were able to determine and express, by the differential calculus, the infinitely small area or volume corresponding to *any* value of the variable, and then, by the integral calculus, to find the sum of *all* such values.

Now, in the case of the pyramid under

consideration, there is no equation, and we must devise some means of expressing the constant relation of the base to any value, x , of the height h . If the base and height of the pyramid be given, as in all practical cases they would be, the necessary data is easily found, because the areas of any cross-sections vary as the squares of their distances from the apex. Hence, if h be the height, and b the area of the base, the area of the base b' corresponding to any distance x from the apex will be:

$$b' = \frac{b x^2}{h^2}$$

Hence:

$$V d = \frac{b x^2}{h^2} dx$$

$$V = \frac{b}{h^2} \int_0^h x^2 dx$$

$$V = \frac{bh}{3}$$

3.—What is the volume of the prolate spheroid, engendered by the revolution of the ellipse about its major axis?

$$y^2 = \frac{B^2}{A^2} (A^2 - x^2)$$

Applying (1), integrating and reducing:

$$V = \frac{1}{3} \pi \frac{B^2}{A^3} (3 A^2 x - x^3) + C.$$

Taking for limits $x = 0$; $x = A$, and doubling the result for the volume of the whole ellipsoid:

$$V = \frac{4}{3} \pi B^2 A$$

This is equal to $\frac{2}{3}$ ds the circumscribing cylinder.

If $A = B$, the ellipsoid becomes a sphere, and $A = B$, becomes radius, R . Hence, for the volume of the sphere:

$$\frac{4}{3} \pi R^3$$

CHAPTER VI.

APPLICATIONS OF THE DIFFERENTIAL AND
INTEGRAL CALCULUS COMBINED.

These applications comprise the greater number of the problems offered for solution, including the rectification of curves and the surfaces of solids, which will now be considered.

PROBLEM.—Let OB (fig. 14) be a curve determined by its equation. It is desired to find the length of the arc $OM = Z$.

Proceeding as in other examples $MM' = dz$ will be the differential of the arc, and we have:

$$dz = (dx^2 + dy^2)^{\frac{1}{2}} \dots (1)$$

EXAMPLE.—Find the length of the cubical parabolic curve $y^3 = ax^2$.

From the equation, $x = \frac{y^{\frac{3}{2}}}{a^{\frac{1}{2}}}$

$$dx = \frac{3}{2} \frac{y^{\frac{1}{2}}}{a^{\frac{1}{2}}} dy; \quad dx^2 = \frac{9}{4} \frac{y}{a} dy^2$$

Substituting in the general equation (1).

$$\begin{aligned} dz &= \left(\frac{9y}{4a} dy^2 + dy^2 \right)^{\frac{1}{2}} \\ &= dy \left(1 + \frac{9y}{4a} \right)^{\frac{1}{2}} \end{aligned}$$

Integrating:

$$\begin{aligned} Z &= \int \left(1 + \frac{9y}{4a} \right)^{\frac{1}{2}} dy \\ &= \frac{8a}{27} \left(1 + \frac{9y}{4a} \right)^{\frac{3}{2}} + C \end{aligned}$$

To find the value of C. When $y = 0$,

$$z = 0. \text{ Therefore, } C = -\frac{8a}{27}.$$

Therefore,

$$Z = \frac{8a}{27} \left(1 + \frac{9y}{4a} \right)^{\frac{3}{2}} - \frac{8a}{27}$$

PROBLEM.—To find the quadrature of the surface of solids of revolution.

Let OB (fig. 14) be the generating curve, and ox the axis of revolution. In revolving MM' the differential of the arc, will describe an infinitely narrow belt, which will be the differential of the surface. The

circumference of the circle of which y is the radius, is $2\pi y$. Since, by previous problem $MM' = z = (dx^2 + dy^2)^{\frac{1}{2}}$, the differential of the surface S of the belt is:

$$dS = 2\pi y (dx^2 + dy^2)^{\frac{1}{2}} \dots (1)$$

EXAMPLES.

1.—Find the surface of the paraboloid, $y^2 = ax$. From the equation:

$$dx^2 = \frac{4y^2 dy^2}{a^2}$$

$$dS = \frac{2\pi}{a} y (4y^2 + a^2)^{\frac{1}{2}} dy$$

$$S = \frac{2\pi}{a} \int y (4y^2 + a^2)^{\frac{1}{2}} dy$$

$$= \frac{\pi}{6a} (4y^2 + a^2)^{\frac{3}{2}} + C$$

$$\text{When } y = 0; c = -\frac{\pi a^2}{6}$$

$$\int = \frac{\pi}{6a} \left\{ (4y^2 + a^2)^{\frac{3}{2}} - a^3 \right\}$$

NOTE.—The student must be warned that all curves cannot be treated as simply as those in the examples. Thus, the common parabola, $y^2 = 2px$ cannot be rectified as readily as the cubical parabola in the above example, but demands an indirect process, beyond the scope of the present volume.

CHAPTER VII.

MISCELLANEOUS APPLICATIONS.

Centres of Gravity ; Spaces passed through by bodies moving with given velocities ; Time required by a vessel containing a fluid to empty itself through an orifice in the bottom, etc.

PROBLEM.—To find the centre of gravity of a plane surface, bounded by a curve.

Conceive the surface to be divided into an infinite number of infinitesimal trapezoids, perpendicular to the axis of symmetry of the figure. These will be the differentials of the area, found in the manner already described. The moment of each, referred to a fixed point on the axis of X , will be its infinitesimal area multiplied by its distance, x , from such point. The moment of the whole area will be equal to the sum of the differentials. This sum, divided by the total area, will give the distance G of the centre of gravity of the

surface, from the given point. The general formula is therefore:

$$G = \frac{\int x \, dA}{\int dA}$$

EXAMPLE.—What is the distance of the centre of gravity of the parabola $y^2 = ax$, from the limiting chord? All constant factors disappear, and we have:

$$G = \frac{\int x^{\frac{3}{2}} \, dx}{\int x^{\frac{1}{2}} \, dx} = \frac{3}{5} x$$

PROBLEM.—To find the centre of gravity of a solid of revolution.

Here it is the differential of the volume of which the moment is taken about the given point and the distance of the center of gravity of the mass from the same point, is found by dividing the sum of such moments by the total volume. Hence

$$G = \frac{\int x \, dV}{\int dV}$$

EXAMPLE.—What is the value of G for the paraboloid $y^2 = ax$?

$$G = \frac{\int x^2 dx}{\int x dx} = \frac{2}{3} x$$

PROBLEM.—What is the space s passed through by a body moving with a given velocity v , under the action of a constant accelerating force?

Let g denote the velocity imparted by the constant force in the unit of time.

Then, in t units of time :

$$g = \frac{v}{t}$$

Since this relation is true for finite values of v and t , it is also true for their differentials, and we have:

$$g = \frac{dv}{dt} \dots \dots \dots (1)$$

In an infinitesimal time, the velocity, though constantly changing, is uniform, and we have:

$$v = \frac{ds}{dt} \dots \dots \dots (2)$$

Hence,

$$ds = v \, dt \dots (3)$$

Inserting the value of dt from (1) in (3);

$$ds = \frac{v \, dv}{g}$$

Integrating,

$$s = \frac{v^2}{2g}$$

PROBLEM.—To find the time a vessel will require to empty itself through an orifice in the bottom.

When a vessel kept constantly full, discharges water through an orifice in its bottom, its velocity (neglecting resistances) is that due to a body falling through a height equal to the depth of water in the vessel. Its velocity is then, putting s for the depth of water:

$$v = \sqrt{2gs}$$

If the area of the orifice be represented by a , and the quantity of water discharged in a given time t , by Q , we have:

$$Q = at \sqrt{2gs}$$

If the vessel, instead of being kept constantly full, is allowed to empty itself, the velocity steadily decreases with the depth of the water. Let A = the area of the descending body of water, which may or may not vary according to the shape of the vessel. In the infinitely short space of time, dt , the volume of discharge is

$$dQ = a dt \sqrt{2gs}$$

During the infinitesimal time the surface of the water has descended through the infinitesimal space ds , therefore, $dQ = A ds$, and we have:

$$A ds = a dt \sqrt{2gs}$$

Whence:

$$dt = \frac{A ds}{a \sqrt{2gs}} \dots \dots (1)$$

EXAMPLE.—How long will a conical vessel of height h , and radius of base r , placed upright with the vertex down, take to empty itself through an orifice of area a , in the vertex?

In this case the area is changing, and at any depth x , it equals $\frac{\pi r^2 x^2}{h^2}$.

From (1) we have, therefore:

$$\begin{aligned}
 t &= \frac{\pi r^2}{a \sqrt{2g} \times h^2} \int \frac{x^2 dx}{x^{\frac{1}{2}}} \\
 &= \frac{\pi r^2}{a \sqrt{2g} \times h^2} \int x^{\frac{3}{2}} dx
 \end{aligned}$$

When $x = h$,

$$t = \frac{2 \pi r^2 \sqrt{h}}{5 a \sqrt{2g}}$$

In the case of a vessel of constant area, such as a cylinder, we do not need to recur to the calculus for a solution of this problem, because the mean velocity of discharge being one-half of that due to the depth of water when the vessel is full, and as the fall of level is regular, we have only to equate the contents of the vessel with the area of orifice into $\frac{gt}{2}$ to get a solution.

The determination of the center of pressure of water against a surface, affords a good example of the application of the calculus.

It will be first necessary to establish the ground work of the calculation.

The total pressure in lbs. upon any plane surface from the water which presses against it is its area in square feet into the distance, in feet, of its center of gravity from the surface of the water, into 62.5, the conventional weight in lbs. of a cubic foot of water. Thus, (fig. 15) the total pressure against the vertical rectangle $ABCD$ from a body of water level with its top AD , is its height into its breadth into half its height into 62.5.

The point of application of this pressure passes through the center of gravity of the volume of water pressing against the given surface, and its direction is normal to such surface. In the figure this volume is the wedge $ABCDEF$, and its centre of gravity, being that of the triangle AEB is situated at one-third the height AB , from the bottom. Calling $AB = h$; AD or $BC = b$, and ignoring the multiplication by 62.5, or rather considering the density of the fluid as = unity, we have:

$$\text{Total pressure, } \frac{h^2 b}{2}$$

Moment of pressure about B C,

$$\frac{h^2 b}{2} \times \frac{h}{3} = \frac{h^3 b}{6}$$

These principles being established, we proceed to the following problem:

PROBLEM.—Find the distance x , of the center of pressure upon the vertical triangular surface, of height h , and base b , standing with its apex up and level with the surface of the water pressing against it.

The center of pressure passes through the center of gravity of the pyramid of water, of base $b h$, and altitude h . To ascertain this, we must first find the area of base, B, at any distance x from the apex. Since the bases are proportional to the squares of the distances from the apex, and the base at the distance, h , being $b h$, we have:

$$B = \frac{b h x^2}{h^2} = \frac{b x^2}{h}$$

The total volume of the pyramid being $\frac{bh^2}{3}$, we have for the distance G of the centre of gravity from the apex:

$$G = \frac{b}{h} \int \frac{x^3 dx}{\frac{bh^2}{3}} = \frac{3}{4} \frac{x^4}{h^3}$$

When $x = h$:

$$G = \frac{3}{4} h$$

The centre of pressure is therefore three-quarters of the height below the surface of the water.

If now we suppose the position of the triangular surface to be reversed, so that the base $A B$ of the triangle $A B C$ (fig. 16) is level with the surface of the water, with the apex c down, the condition of things is entirely changed, so that we cannot get the height of the centre of gravity of the pyramid $A B C D$ in the same way as in the previous example. We must find the sum of the moments of all the infinitesimal triangles $a b c$, etc., about the axis

D C, and divide (as before) by the volume of the pyramid. We proceed thus: The area,

a , of any small triangle $a b c$ is $\frac{A x^2}{h^2}$, A

being the area of the triangle A B C, distant D C = h , from the vertex D, and x being the distance of the triangle $a b c$ from the same point. The moment of the

infinitesimal volume $\frac{A x^2 dx}{h^2}$ about the

line D C is therefore $\frac{x^2 A dx}{h^2} \times \frac{2}{3}x$, and the

sum of the moments of all such volumes is

$\frac{2 A}{3 h^2} \int x^3 dx = \frac{A x^4}{6 h^2}$. When $x = h$,

this becomes $\frac{A h^2}{6}$, and the distance G of

the centre of pressure is:

$$G = \frac{\frac{A h^2}{6}}{\frac{A h}{3}} = \frac{h}{2}$$

It will be instructive to see how these results may be reached by algebraic methods. Let A B C, B C D, (fig. 17) be the

triangle in its two positions. No matter what its shape, it may be brought into the form shown, merely keeping the base and height as given. The sum of the moments of the pressure upon the two triangles A B C, B C D about C D must be equal to that upon the rectangle A B C D about the same line. The moment of the pressure upon the rectangle, of area A, we know to be $\frac{A h}{2} \times \frac{h}{3} = \frac{A h^2}{6}$. The moment of the pressure upon the triangle A B C about C, is equal to its area into the distance of its centre of gravity below the surface of the water, into the sought distance, y , of the centre of pressure of the water. It is, therefore, $\frac{A}{2} \times \frac{h}{3} \times y = \frac{A h y}{6}$. In the same way, the moment of the pressure upon the triangle B C D about C D, is found to be $\frac{A h x}{3}$.

Equating, we have:

$$\frac{A h y}{6} + \frac{A h x}{3} = \frac{A h^2}{6}$$

$$\text{Whence, } y = h - 2x \quad . \quad . \quad . \quad (1)$$

We now want another equation, in order to eliminate y . This is furnished by the noteworthy consideration, that the area of the two triangles being equal, their moments about CD are equal; and since the total pressure upon them is equal to the distances of their centres of gravity below AB , the lever-arms x and y , must be inversely proportional to the distances. Therefore:

$$\frac{x}{y} = \frac{\frac{h}{3}}{\frac{2h}{3}} = \frac{1}{2}. \quad \text{Whence:}$$

$$y = 2x \quad . \quad . \quad . \quad . \quad (2)$$

Comparing (1) and (2)

$$h - 2x = 2x$$

$$x = \frac{h}{4}$$

and,

$$y = \frac{h}{2}$$

This investigation is very instructive, and is illustrative of the fact that it will

generally, though not always, be found, in questions of practical mathematics, that the calculus is a convenience rather than a necessity.

The above are a few only, of the many problems of applied mechanics that can be solved by means of the elementary portion of the calculus contained in the preceding pages. Other more intricate questions, particularly those of astronomy, require the most advanced resources of the science for their solution.

CHAPTER VIII.

LIMITATIONS OF THE CALCULUS, AND
DIFFICULTIES PRESENTED BY THE CONCEPTIONS
UPON WHICH IT IS BASED.

In the preceeding chapters, the most elementary features only of the calculus have been developed. Our investigation have been limited to the consideration of the first differentials of algebraic functions of an independent variable and their corresponding integrals. Beyond this narrow field lies a vast region which we have left unexplored. But even these small beginnings have led to the solution of many beautiful and interesting problems, some of which have possessed a high degree of practical utility.

It must be borne in mind however, that the true province of the calculus commences where that of algebra and analytical geometry ends. It was devised to enable investigators of the profoundest laws of nature to solve problems which

stubbornly resisted the action of the mathematical solvents already at their command. By its aid transient, instantaneous values of constantly fluctuating functions are caught in the act, and by ratios established between far less than microscopic atoms, the motions of the heavenly bodies, separated as they are by vast interstellar spaces, can be accurately determined.

This is as much as to say that its practical use in every day calculations is very restricted. If eggs are worth one cent a piece, the calculus does not help us in the least to ascertain how much they are a dozen. But if one of the eggs were launched into space with a given velocity, the calculus would greatly aid us in calculating the tangent to the curve which it described.

The reader will have remarked that in all the problems relating to tangents, areas, and surfaces of curves and the volumes engendered by their revolution, it has always been necessary to know the equation to the curve, in order to apply

the methods of the calculus. It is powerless to give the area of a figure bounded by random lines. Such area must be ascertained by mechanical means, notably by the aid of the planimeter—that wonderful mechanical integrator that refuses no figure however erratic. The mind must therefore be disabused of the idea that the calculus furnishes a short cut to general solutions. To repeat what has been already stated, its use is mostly confined to the exigencies of scientific research, with the notable exception of the determination of maximum and minimum values.

Turning from the remarkable results achieved by the calculus, to the mathematical bases upon which it rests, the algebraist cannot fail to find them at first sight somewhat unsatisfactory. He is confronted at the start by the curious anomaly of an exact science built apparently upon a foundation of approximations. The reasons given for eliminating the higher degrees of the differentials, for instance, seem to him specious, not to say artful, particularly when, as in the case of the rec-

tification of curves, he sees that differentials are freely squared. He may even venture upon the perilous ground of endeavoring himself to find some more rational method of eliminating superfluous terms than that of simply ignoring their existence, but he will find that though he may succeed in some special cases, he is unable to generalize his methods. On the other hand, the more thought he gives to the apparent anomalies and mathematical fallacies which at the outset so disturbed him, the less irrational they appear. He sees that the discarding of the square of the differential in one case, and using it in another, rests upon perfectly logical grounds, and finally, the triumphant success which attends the application of rules thus founded, disarms his opposition, and renders him indisposed to cavil at the reasoning upon which these splendid results are based, particularly when this reasoning commends itself to his understanding the more closely he studies it. As he advances further and further in the science he will be disposed to sink all other feelings re-

garding it in admiration at the way in which by its aid, to use the words of the late Professor William Smyth—"Problems which, solved by the previous methods, not excepting the more effective ones of Descartes, were tedious and embarrassing, are solved by it with the greatest facility; and others, the solution of which entirely baffled the skill of the ancient geometers, yield at once to its powers."

Fig. 4

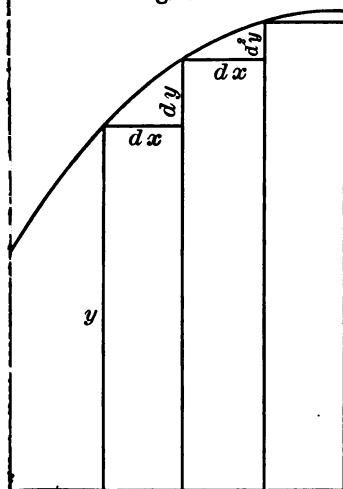
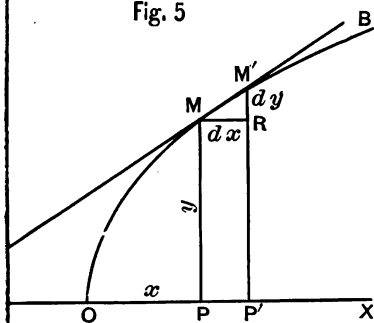


Fig. 5



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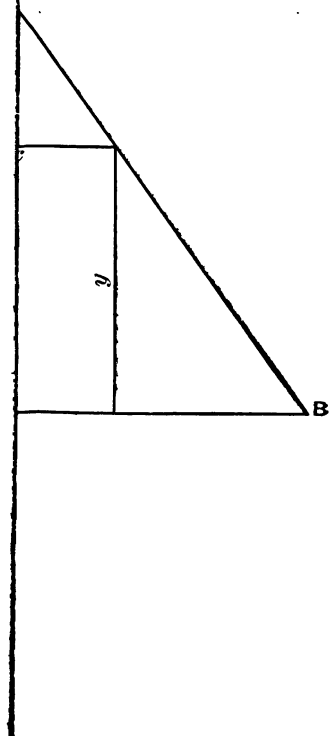
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Fig. 7





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Fig. 8

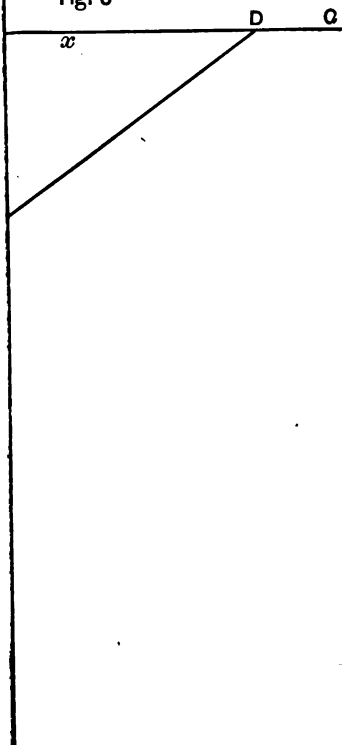
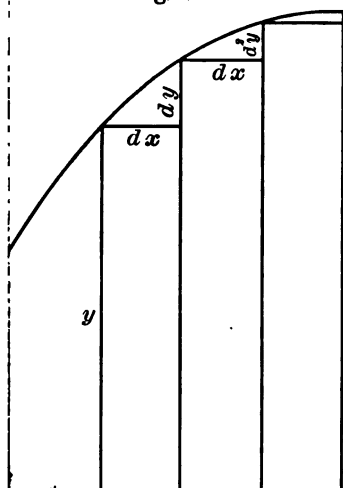


Fig. 4



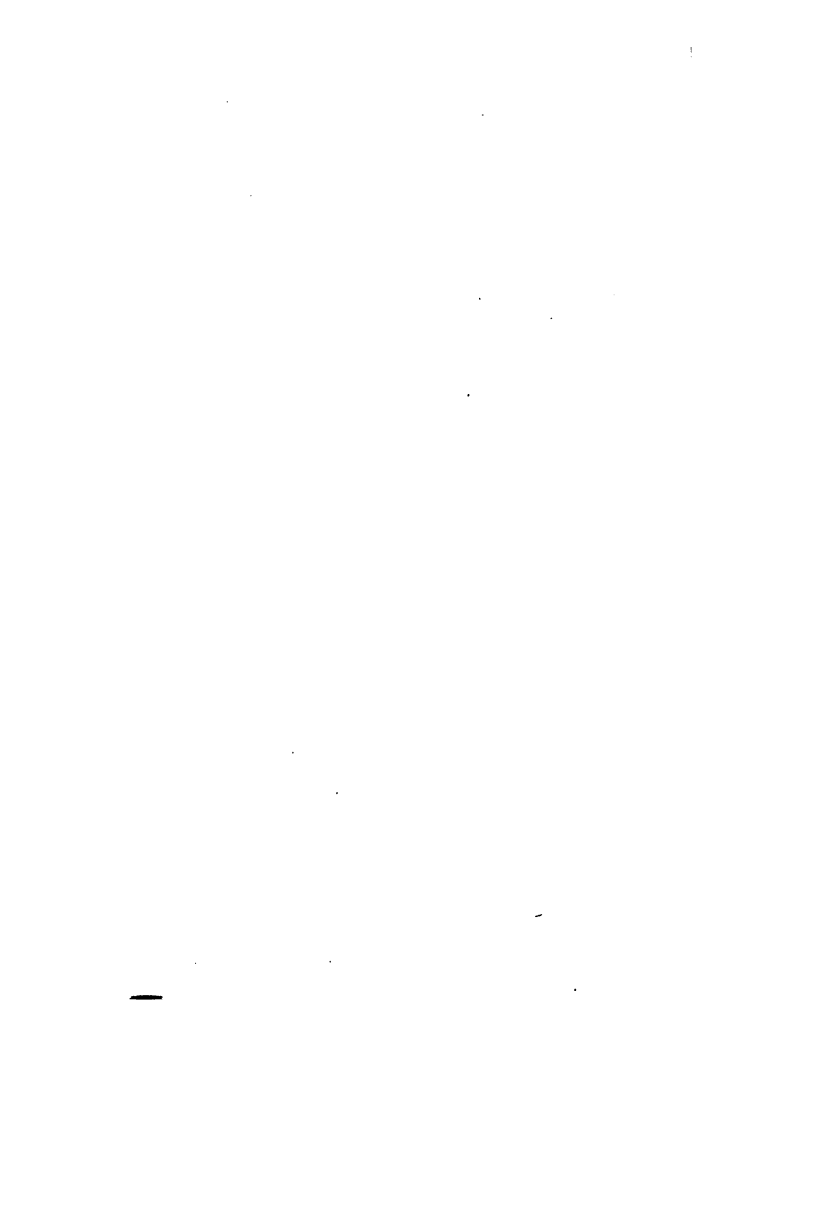
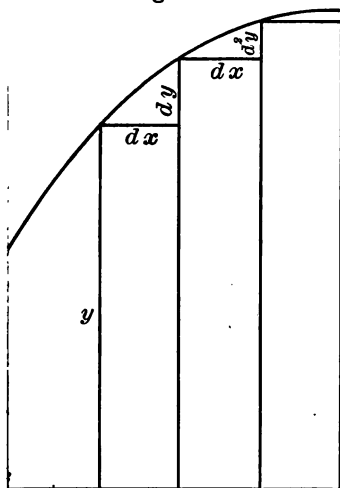
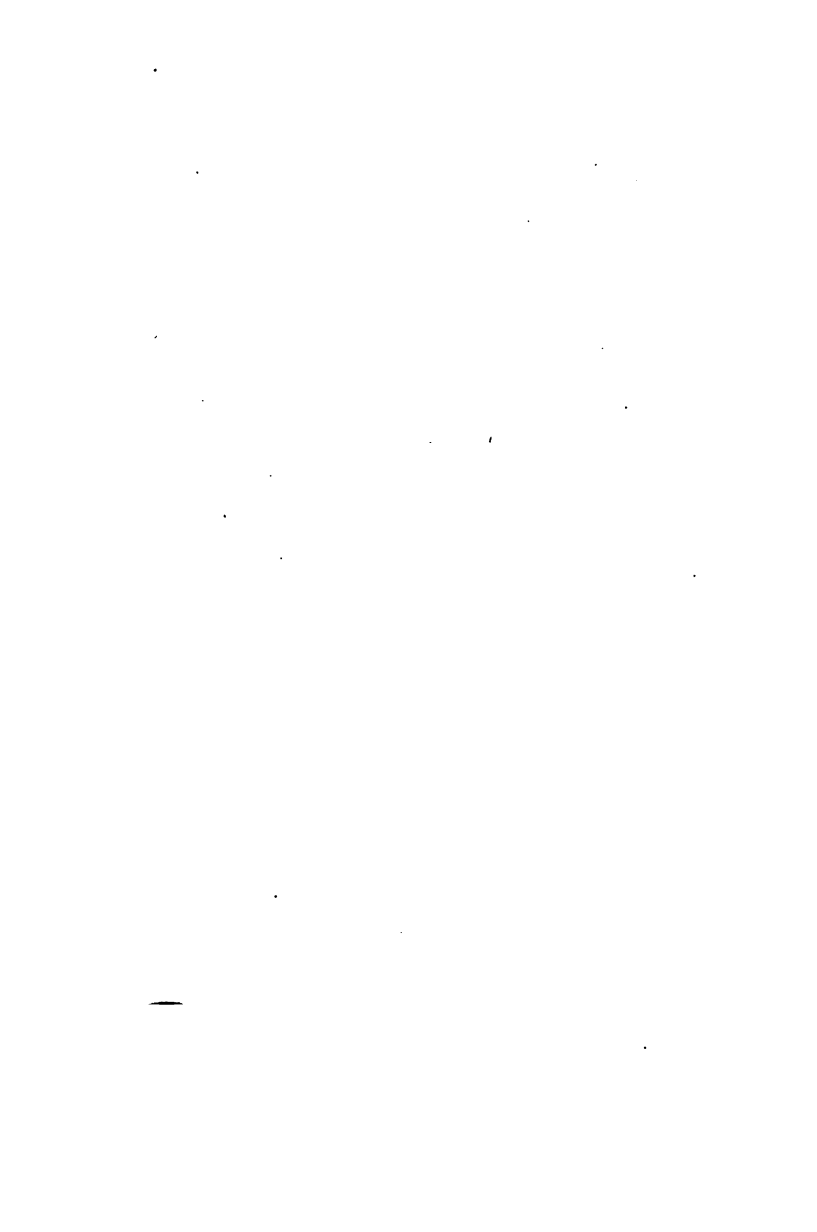


Fig. 4







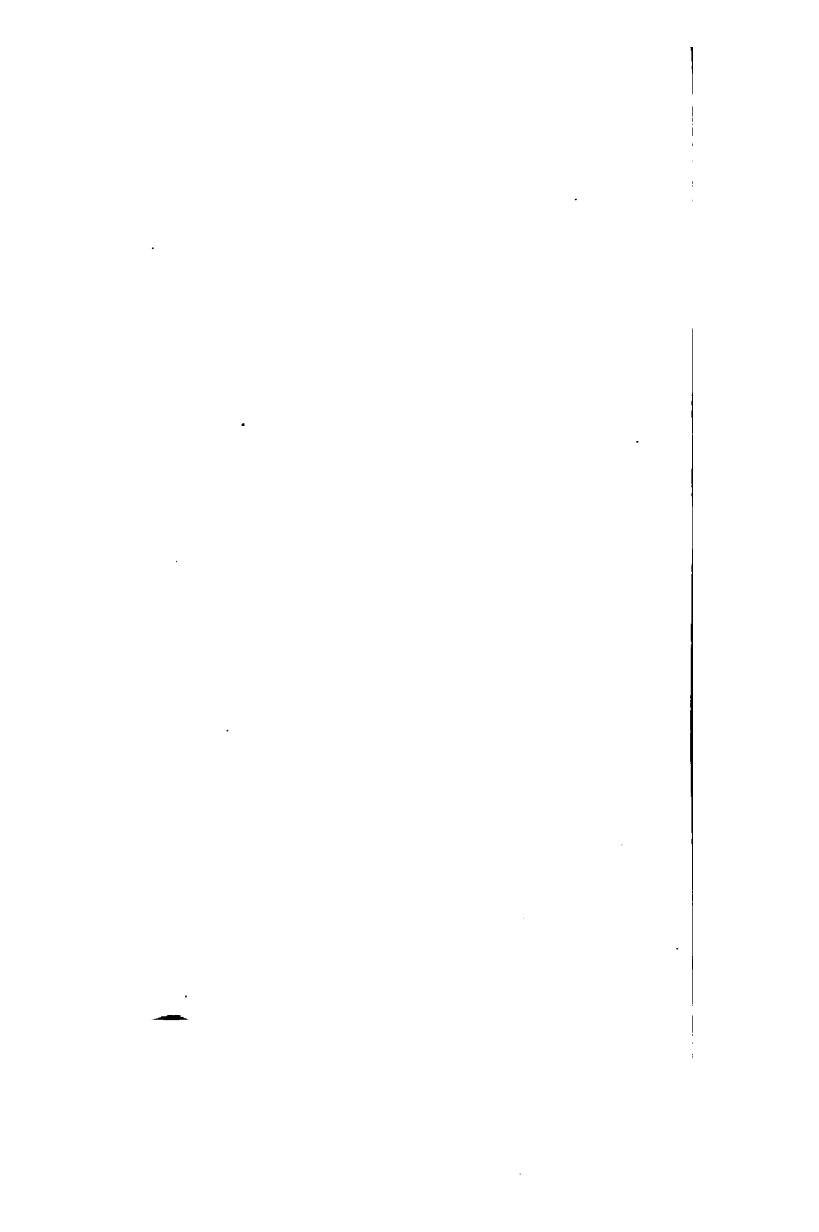


Fig. 6

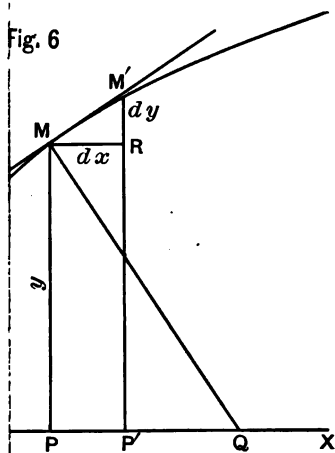


Fig. 7

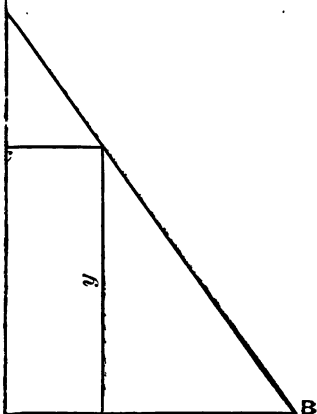
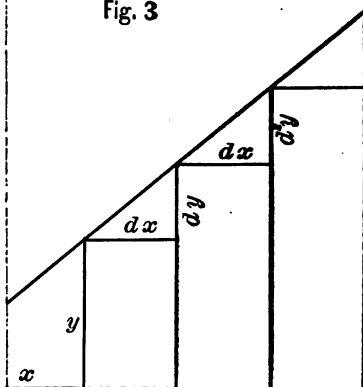




Fig. 3



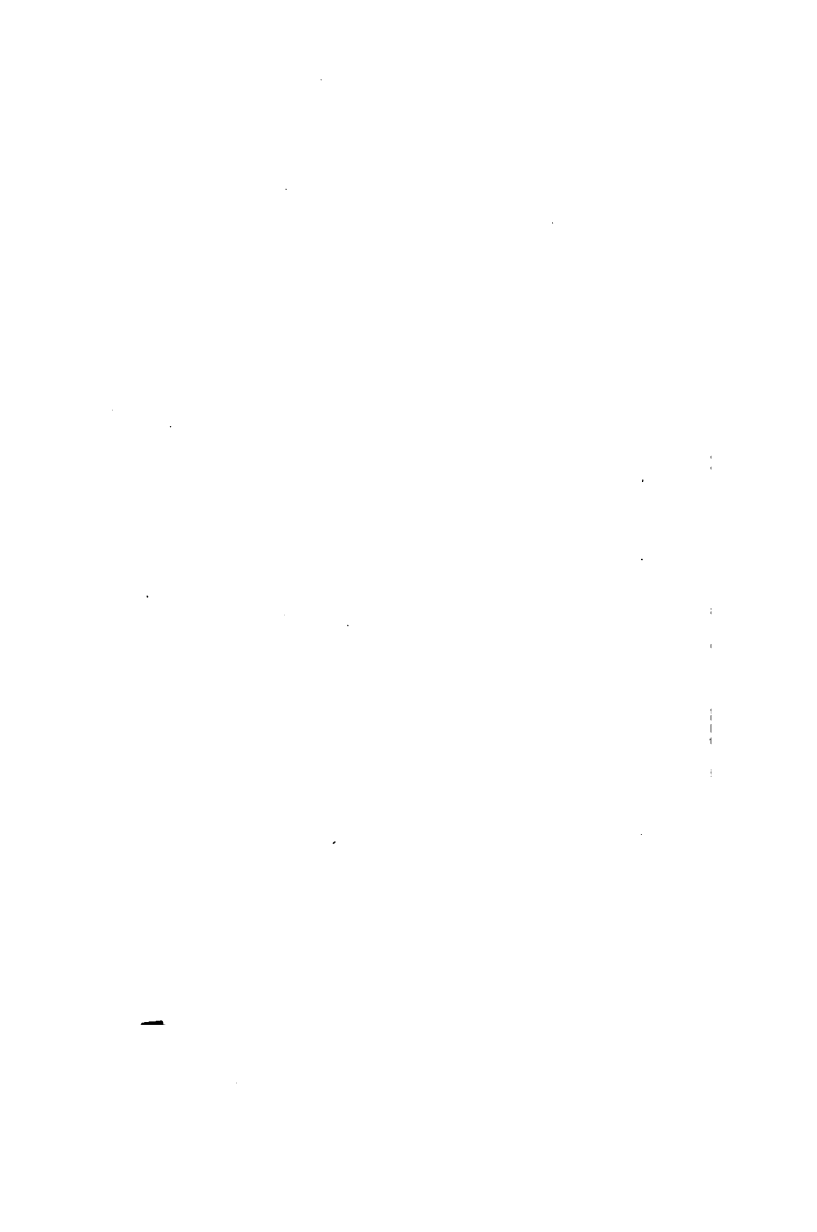
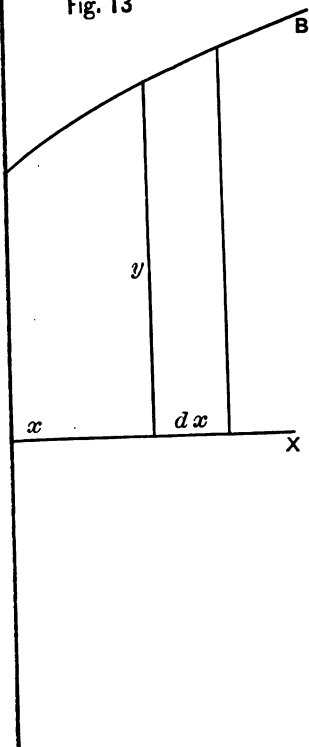


Fig. 13



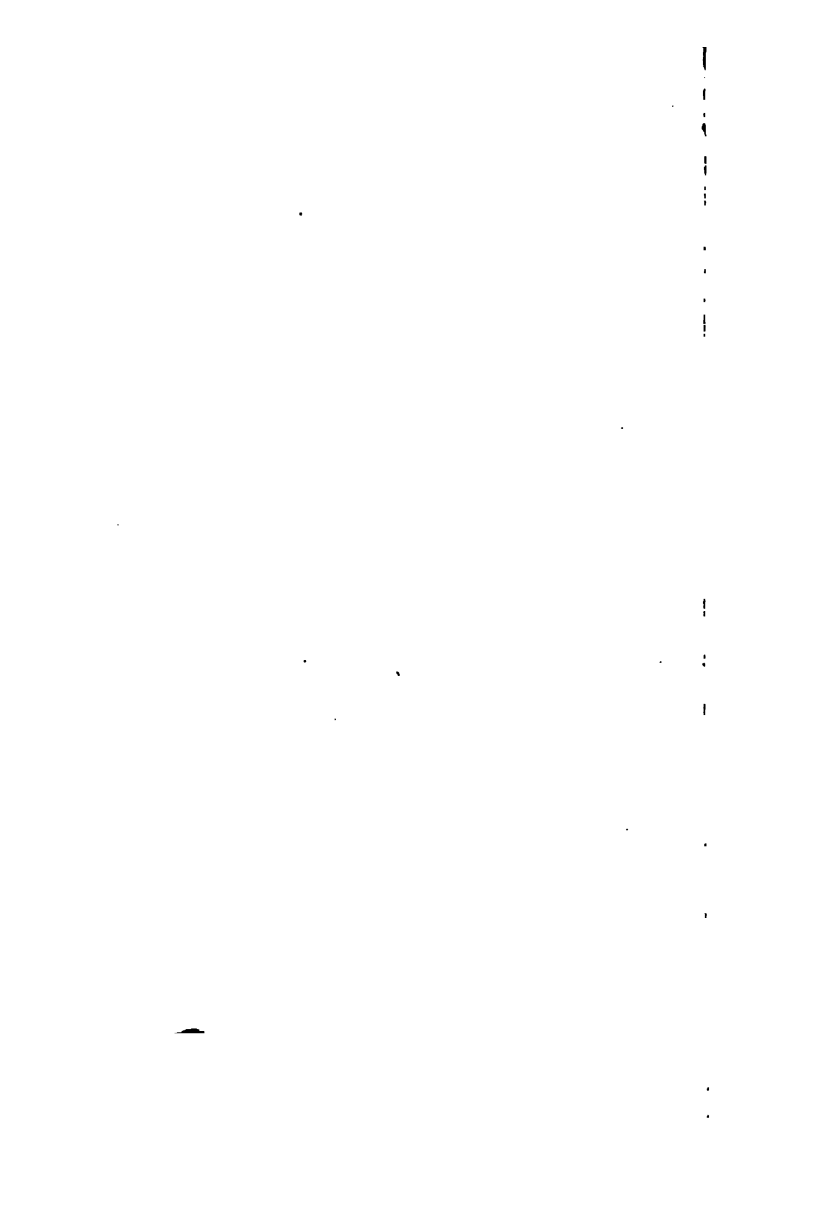


Fig. 14

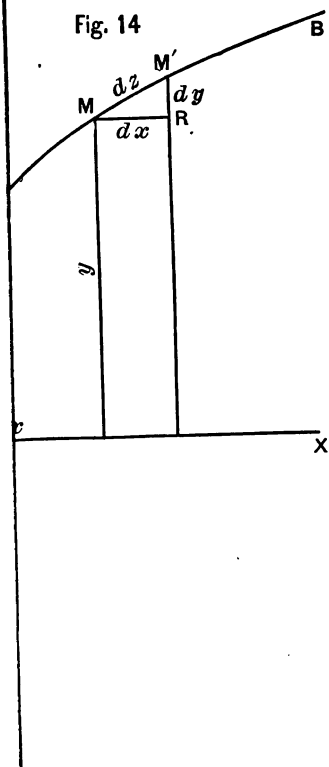
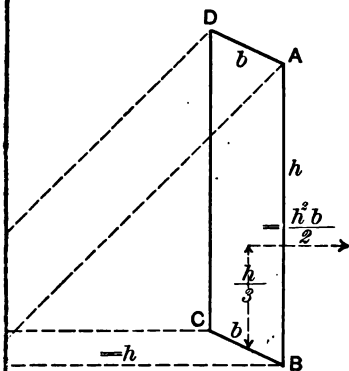
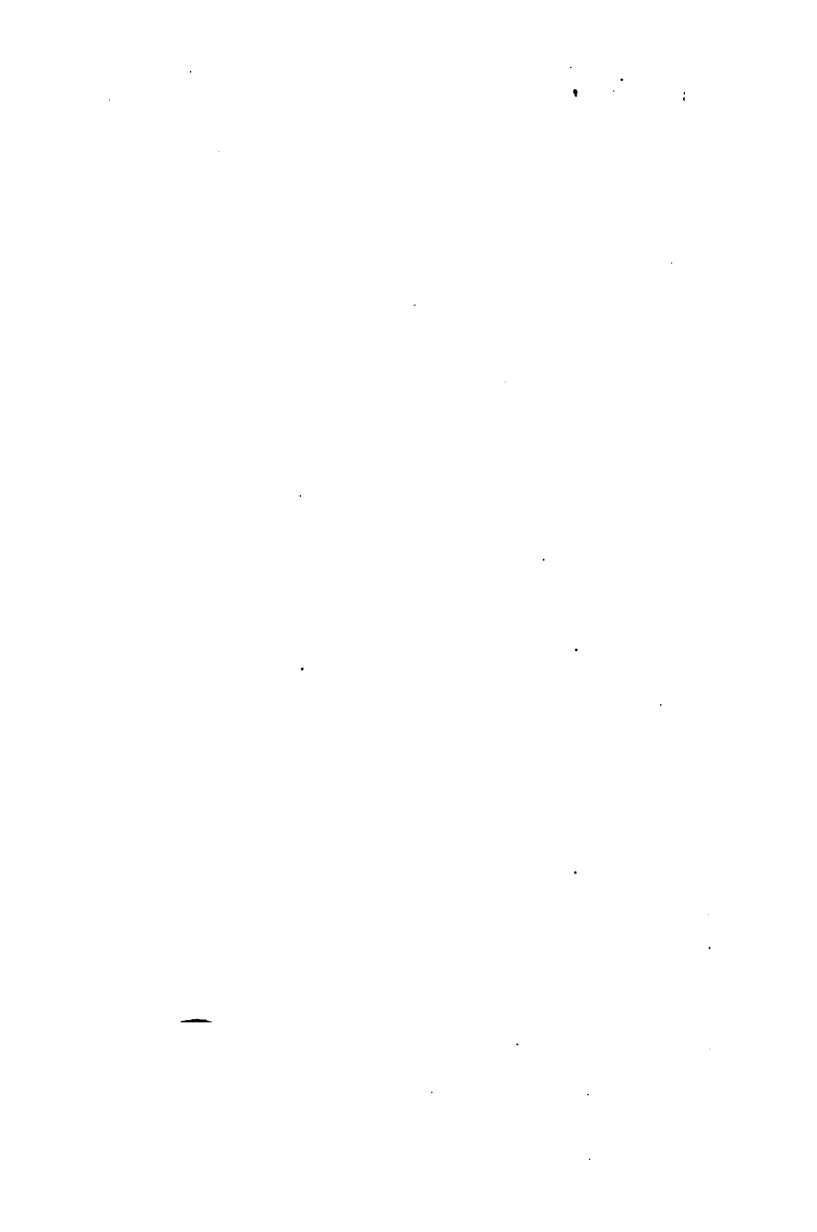




Fig. 15





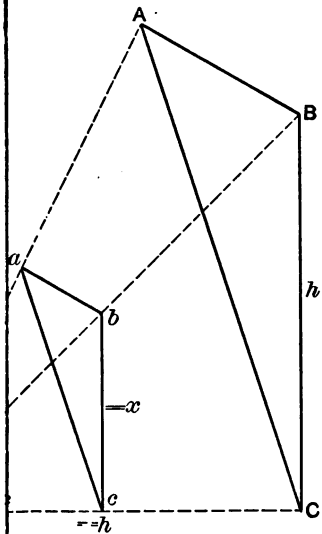
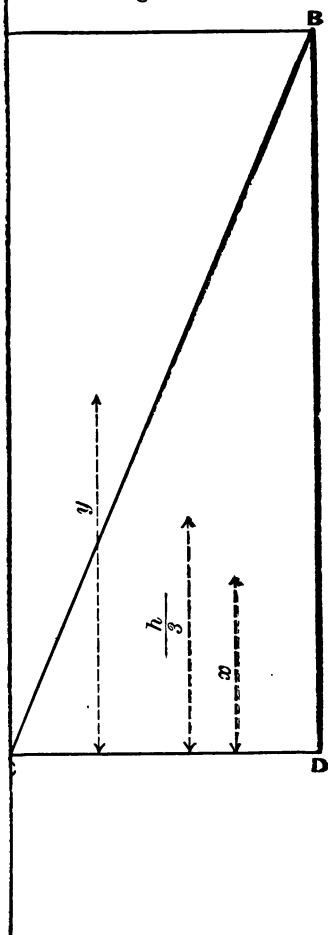
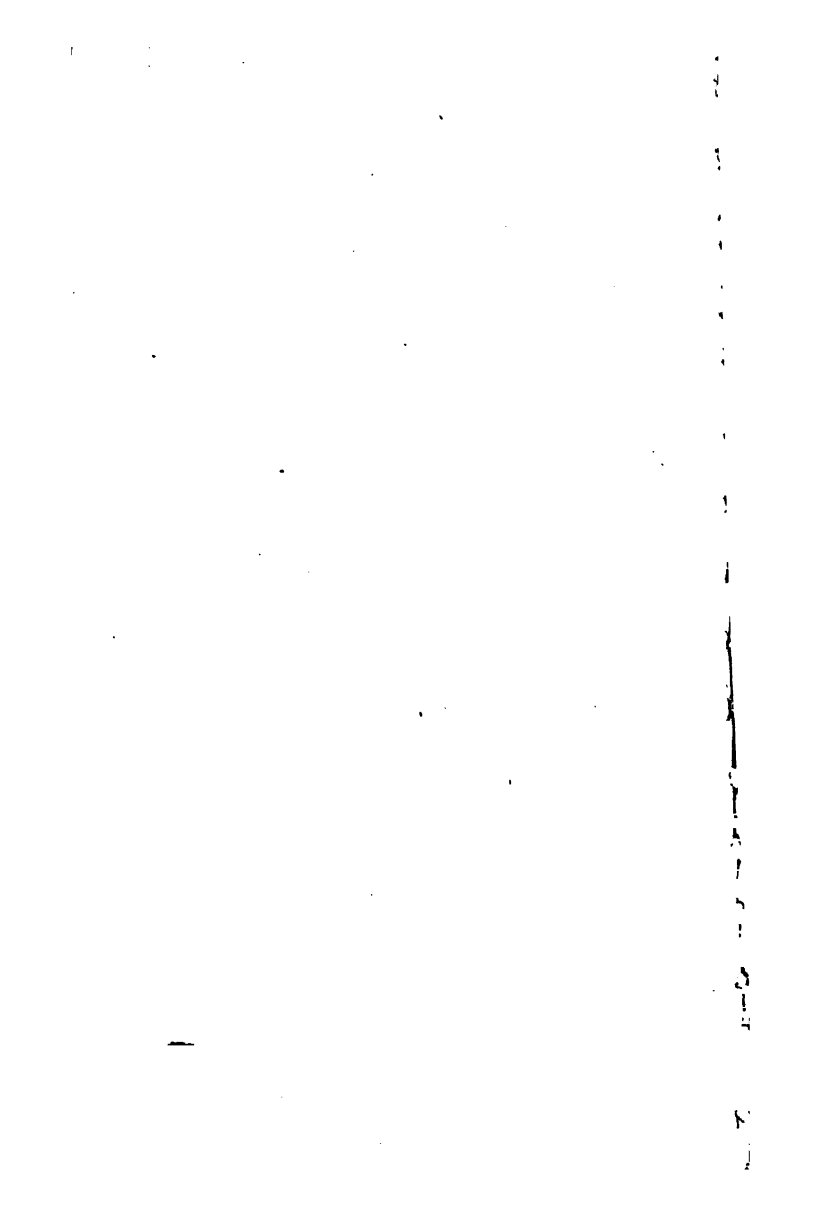
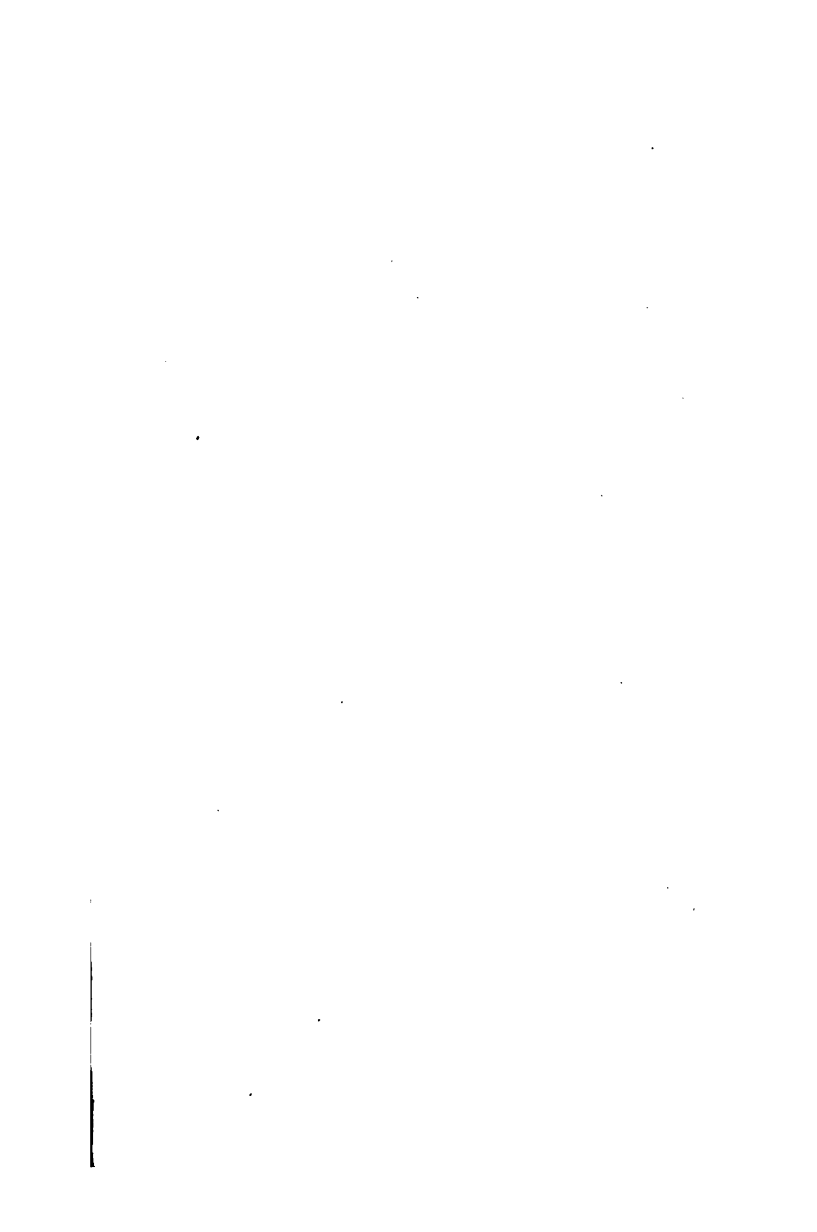




Fig. 17











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